Yashwantrao Chavan College of Science Karad
B.Sc III : Semester V : Paper IX
Subject: Mathematical Physics
Question Bank

Select the correct alternatives for each of the following

(i) Every partial differential equation involves at least how many independent variables?

a) 1
b) 2
c) 3
d) none of these

(ii) The order and degree of the differential equation $\frac{\partial^2 z}{\partial x^2} = k\frac{\partial z}{\partial y}$ is:

a) 1, 2
b) 1, 1
c) 2, 1
d) 2, 2

(iii) The equation $f(x) + g(y) = 1$ is called a

$_{equation.}$

(iv) Laplace

(v) Linear

(vi) Non-linear

(vii) Heat

Which of the following best describes the Gamma function?

a) A function used primarily in probability theory to model the distributions of continuous random variables.
b) A function used to compute the area under the normal distribution curve.
c) A function that extends the factorial function to non-integer arguments.
d) A function used to calculate the error function.

The Beta function is defined as:

a) $\beta(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$
b) $\beta(x, y) = \int_0^1 t^{x}(1-t)^{y}dt$
c) $\beta(x, y) = \int_0^\infty t^{x-1}e^{-t}dt$
d) $\beta(x, y) = \int_0^x t^{y-1}e^{-t}dt$

The relation between Beta and Gamma functions is given by:

a) $\beta(x, y) = \frac{\Gamma(x)\cdot\Gamma(y)}{\Gamma(x+y)}$
b) $\beta(x, y) = \frac{\Gamma(x+1)\cdot\Gamma(y+1)}{\Gamma(x+y+1)}$
c) $\beta(x, y) = \Gamma(x) + \Gamma(y)$
d) $\beta(x, y) = \Gamma(x) - \Gamma(y)$

The integral $\int_0^\infty e^{-t^2}dt$ evaluates to:

a) $\sqrt{\pi}$
b) $2\sqrt{\pi}$
c) $\frac{\sqrt{\pi}}{2}$
d) $\frac{\sqrt{\pi}}{4}$

The Error function has the following limit as x approaches infinity:

a) $\lim_{x \to \infty} erf(x) = 1$
b) $\lim_{x \to \infty} erf(x) = -1$
c) $\lim_{x \to \infty} erf(x) = 0$
d) $\lim_{x \to \infty} erf(x) = \infty$

The relationship between the Error function and the complementary Error function is:

a) $erf(x) + erfc(x) = 0$
b) $erf(x) + erfc(x) = 1$
c) $erf(x) \cdot erfc(x) = 1$
d) $erf(x) - erfc(x) = 1$

The Error function is used to compute:

a) The area under the exponential distribution curve.
b) The area under the normal distribution curve.
c) The area under the uniform distribution curve.
d) The area under the beta distribution curve.

Complex numbers are typically represented in the form:

a) $a + bi$
b) $a - bi$
c) $a \cdot bi$
d) $a \div bi$

The graphical representation of a complex number involves which of the following:

a) A point in the Cartesian plane
b) A line segment
c) A curve
d) A vector in a polar coordinate system

Equal complex numbers have:

a) The same real part
b) The same imaginary part
c) Both the same real and imaginary parts
d) None of these

The result of adding two complex numbers is obtained by:

a) Adding their real parts and imaginary parts separately
b) Subtracting their real parts and imaginary parts separately
c) Multiplying their real parts and imaginary parts separately
d) Dividing their real parts and imaginary parts separately

Multiplication of two complex numbers is equivalent to:

a) Distributing one complex number over the other
b) Taking the product of their magnitudes and adding their arguments
c) Taking the product of their magnitudes and subtracting their arguments
d) Taking the product of their magnitudes and multiplying their arguments

The division of two complex numbers is achieved by:

a) Rationalizing the denominator
b) Finding the reciprocal of the divisor and multiplying
c) Finding the square root of the dividend and divisor
d) Finding the square root of the dividend and multiplying

A complex number z such that $z^2 = -1$ is:

a) A real number

b) An imaginary number

c) A complex number with both real and imaginary parts

d) Undefined

The principal square root of a complex number z is:

a) $\sqrt{z} = \sqrt{r}(\cos(\theta/2) + i\sin(\theta/2))$

b) $\sqrt{z} = \sqrt{r}(\cos(\theta) + i\sin(\theta))$

c) $\sqrt{z} = \sqrt{r}(\cos(2\theta) + i\sin(2\theta))$

d) $\sqrt{z} = \sqrt{r}(\cos(2\theta) - i\sin(2\theta))$

The Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ is often used to represent:

a) Trigonometric functions

b) Exponential functions

c) Logarithmic functions

d) Polynomial functions

De Moivre's theorem states that for any complex number $z = r(\cos\theta + i\sin\theta)$ and integer n, $(\cos\theta + i\sin\theta)^n$ is equal to:

a) $r^n(\cos(n\theta) + i\sin(n\theta))$

b) $r^n(\cos(\theta/n) + i\sin(\theta/n))$

c) $r^n(\cos(n\theta) - i\sin(n\theta))$

d) $r^n(\cos(\theta/n) - i\sin(\theta/n))$

The Cauchy-Riemann conditions describe the conditions for:

a) Differentiability of complex functions

b) Integrability of complex functions

c) Continuity of complex functions

d) Monotonicity of complex functions

Short and Long Answer Type Questions

1. Explain the properties and significance of the Gamma function in mathematical analysis.

2. Define the Beta function and discuss its applications in mathematics and statistics.

3. Describe the relationship between the Beta and Gamma functions, providing examples where both functions are used.

4. Explain the Error function and its role in probability theory and mathematical analysis.

5. Solve the integral $\int_0^x e^{-t^2}\,dt$ using the Error function, and discuss its importance in mathematical modeling.

6. Define the Gamma function and explain its properties.

7. Describe the Beta function and discuss its properties. How is it related to the Gamma function?

8. What is the Error function and how is it related to the Gamma function? Explain its significance.

9. State Euler's formula and discuss its implications in representing complex numbers.

10. Explain De Moivre's theorem and provide an example illustrating its application.

Problems for Practice

1. Solve the integral equation $2\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial t^2} = 0$ by the method of separation of variables with boundary conditions $u(0,t) = u(3,t) = 0$ and $u(x,0) = 5\sin(4x)$. [Ans.: $u(x,t) = 5\sin(4x)e^{-3t^2\pi^2}$]

2. Solve the equation $\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial t^2} = 0$ by the method of separation of variables.

3. Solve the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ by the method of separation of variables.

4. Solve the equation $x^2\frac{\partial^2 u}{\partial y^2} + t^2 = 0$ by using the method of separation of variables.

5. Solve the integral $\int_{-\infty}^{\infty} e^{-x^2} dx$ using appropriate techniques, and discuss its importance in probability theory.

(i) Solve the equation
$\frac{\partial^2 u}{\partial x^2} + 1/2\frac{\partial^2 u}{\partial t^2} = 0$ by the method of separation of variables with boundary conditions $u(0,t) = u(3,t) = 0$ and $u(x,0) = 5\sin(4x)$.

(ii) Solve the equation
$\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial t^2} = 0$ by the method of separation of variables.

(iii) Solve the equation
$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$ by the method of separation of variables.

(iv) Solve the equation
$x^2\frac{\partial^2 u}{\partial y^2} + t^2 = 0$ by using the method of separation of variables. **(v)** Compute the principal square root of the complex number $z = 3(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$.
(vi) Use De Moivre's theorem to find the cube roots of the complex number $z = 8(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$.
(vii) Verify the Cauchy-Riemann conditions for the function $f(z) = x^2 - y^2 + i(2xy)$, where $z = x + iy$.
(viii) Find the value of the integral $\int_0^{\infty} e^{-x^2} dx$ using the properties of the Error function.
(ix) Solve the differential equation $\frac{d^2y}{dx^2} + 4y = 0$ using Euler's formula and De Moivre's theorem.