B.Sc. (Part-III) (Semester -V) Examination
MATHEMATICS
Abstract Algebra (DSE E-10)
Sub. Code: 79673
Question Bank

Q. Select the correct alternative of the following

1) Which of the following is not group.

a) (Z, +)   b) (R, +)   c) (R , ·)   d) (Zn , +)

2) Order of group is equal to……………..

a) Number of element in group G   b) Finite

c) Number of element in subgroup G   d) infinite

3) Inverse of ω^2 in a group {1, ω, ω^2} w.r.t. multiplication where ω^3=1 is…………

a) 1   b) ω c) ω^2 d) ω^3

4) Order of Rotation in Dihedral group D_5 is…………………

a) 4   b) 10   c) 5   d) 2

5) Order of Reflection in any Dihedral group D_n is…………………

a) n   b) 1   c) 2n   d) 2

6) In Dihedral group D10 number of rotation is 10 then number of reflection is equal to…………………

a) 5   b) 10   c) 20   d) 2

7) Which is not example on Symmetric group.

a) D_4 b) S_3 c) (R , +)   d) A_3

8) In symmetric group S_3 & S_4 number of element is equal to……… &…….respectively

a) 2 ,12   b) 4 ,24   c) 6 , 24   d) 6 , 12

9) Which is not an abelian group.

a) (Z ,+) b) (R ,+) c) S_3 d) D_2

10) Dihedral group D_4 & Symmetric group S_3 are…………………….

a) Finite & abelian b) Finite & non abelian

c) Infinite & abelian d) infinite & non abelian

11) Cycle length of ……… is called Transposition.

a) 2 b) 4 c) 1 d) 3

12) Consider the Statement:

I) F = (1 2 3 4 5) is even permutation

II) F = (1 2 3 4) is odd permutation.

a) Only I is true b) only II is true c) Both I & II are true d) both I & II are not true

13) G'= {e} if & only if G is ………………….

a) Abelian b) non abelian c) Cyclic d) non cyclic

14) If O(G)= 49 then derived subgroup of G then G' =...........

a) G b) G' c) {e} d) None of these

15) If G= S_3 then G'=…………………..

a) D_4 b) S_3 c) S_2 d) A_3

16) If H is subgroup of group G with index 2 then H is ....In G.

a Abelian b) normal c) Non abelian d) simple

17) The union of all conjugate Class is equal to ………………..

a) e b) G c) cl (a) d) Z(G)

18) If O(G)= 27 & G is non abelian then no of conjugate class in G ....

a) 11 b) 12 c)10 d) 24

19) let G be a non abelian group order P3 then Z(G) =………

a) G b) e c) G' d) P

20)…………is not a ring in the following structure.

a) (R ,+ , ·) b) ($\boldsymbol{N}$,+ , ·) c) (Q ,+ , ·) d) (C ,+ , ·)

21) For a ring R ------

a) there is left identity b) there is right identity

c) both a) & b) d) none of them

22) Let R be a ring then R is called commutative ring if --------

a) a + b= b + a ∀ a ,b ∈ R b) a · b= b · a ∀ a ,b ∈ R

c) Both a) & b) d) none of them

23) Which of the following is not integral domain -------

a) (R ,+ , ·) b) ($\boldsymbol{N}$,+ , ·) c) (Z_4 ,+ , ·) d) (C ,+ , ·)

24)…………is an integral domain in the following.

a) (Z × Z ,+ , ·) b) ($\boldsymbol{Q}$,+ , ·) c) (Z_4 ,+ , ·) d) M_2(R)

25) An element a in a ring R such that a2=a is known as…………element.

a) Identity b) nilpotent c) Idempotent d) unity

26) Let R be a ring, an element 0 ≠ a ∈ R is called zero divisor if there is

0 ≠ b ∈ R such that ___________

a) ab = a b) ab = b c) ab = 0 d) ab ≠ 0

27) A ring R such that, x^2 = x ∀ x Є R is called as ___________

a)Division Ring b) idempotent element c) Boolean ring d) Nilpotent element

28) If A & B is two ideals of ring R then A + B=…………………

a) A∪B b) A ∩ B c) < A∪B> d) < A ∩ B>

29) Let R be a ring if their exist positive number n such that,

Ch R = n iff na = 0 ∀ a Є R is called as __________

a) Zero divosors b) characteristics of ring

c) Simple ring d) integral domain

30) For the ring of integer modulo 5 Characteristics of Z_5 is ……..

a) 1 b) 3 c) 4 d) 5

31) If 1 is of additive order n then characteristics of R is..........

a) 1 b) n c) ∞ d) 0

32) if D is an integral domain then characteristics of D is……………

a) Zero b) Prime c) Zero or prime d) Infinite

33) A ring R having no proper ideals, then ring R is called……ring

a) Division b) Quotient c) Simple d) commutative

34) A division ring is a ____________

a) Simple ring b) integral domain c) Quotient ring d) commutative ring

35) For the ring (Z,+,.) the mapping f: Z→Z defined by f (n) = 2n, $\forall n \epsilon Z$ is…………..

a) Not a homomorphism b) Epimorphism

c) Endomorphism d) Monomorphism

36) Consider the Statement:

I) A onto homomorphism is called Epimorphism

II) A one one homomorphism is called Monomorphism

a) Only I is true b) only II) is true

c) both I & II is true d) both I & II is not true

37) Let f : R → R' is homomorphism then, f(0) = ………

a) 0 b) 0' c) e d) e'

38) Let f: R → R' is ring homomorphism then the ker f is an …………of R

a) Left ideal b) Right ideal c) Ideal d) Coset

39) An ideal M of commutative ring R with unity is maximal ideal of R if and only If RM is…………….

a) Field b) a skew field c) Prime ideal d) Comaximal

40) Any ring can be imbedded into…………

a) Ring without unity b) Ring with unity

c) Ring with zero divisors d) Ring without zero divisors

Q. Long Answer Type Questions

1. If f: R → R' is an onto homomorphism then prove that R' is isomorphic to a quotient ring of R.

2. State and prove Fundamental theorem of Ring homomorphism

3. Let R be a commutative ring. Prove that an ideal P of R is prime if and only if R/P is an integral domain.

4. Define characteristic of a ring. If D is an integral domain then prove that characteristic of D is either zero or a prime number.

5. If R is a commutative ring with unity then prove that an ideal M of R is maximal ideal of R if and only if R/M is a field.

6. State and prove Lagrange's Theorem.

7. If A,B are two ideals of a ring R then prove that A+B / A ≅ B/ A ∩ B

8. Let G' be the commutator subgroup of a group G then prove that (i) G' is normal in G (ii) G/G' is abelian (iii) G' is the smallest subgroup of G such that G/G' is abelian.

9. Define conjugates. Prove that cl (a) = {a} iff a $\in$ Z(G), where cl (a) is conjugacy class of a in G and Z(G) is a centre of G.

10. Show that G/N is isomorphic to the multiplicative group {1, -1}, where G = D_{2n} and N = $\{y, y^2, \ldots y^{n-1}, y^n = e\}$.

11. Let ={ ±1, ± i, ±j, ± k}. Define product on G by usual multiplication with $i^2 = i^2 = i^2 = -1$, i.j = -j.i = k, j.k = -k.j = I, k.i = -i.k = j then show that G is a group.

12. In a ring R prove that (i)a.0=0.a=0, (ii)a(-b)=(-a)b= -ab, (iii) (-a)(-b)=ab, (iv) a(b-c) = ab – ac,

13. Let R[x] be the Ring of polynomials over a ring then prove that

(i) R is commutative iff R[x] is commutative and (ii)R has unity iff R[x] has unity.

14. State and prove that First theorem of isomorphism.

15. State and prove that Second theorem of isomorphism.

Short Answer Type Questions

1. If A and B are two ideals of ring R then prove that A+B is an ideal of R containing both A and B.

2. Prove that a homomorphism f: G → G' is one-one if and only if Kerf= {e}.

3. If f: R → R' is a homomorphism then prove that Kerf is an ideal of R.

4. Prove that every finite integral domain is a field.

5. If D is an integral domain, then prove that characteristic of D is either zero or a prime number.

6. State and prove Fermat's Theorem.

7. If R be a ring with unity and 1 is additive order n then ChR = n. If 1 is of additive order infinity then prove that ChR = 0

8. Show that intersection of two ideals is an ideal.

9. Define a simple ring. Prove that division ring is simple ring.

10. If L is a left ideal of a ring R and $\lambda(L) = \{x \in R / xa = 0$ for all $a \in L\}$ then show that $\lambda(L)$ is an ideal of R.

11. State and prove Euler's Theorem.

12. Prove that a homomorphism f: G → G' is one-one iff kerf = {e}.

13. If A and B of two ideals of ring R then prove that A+B is an ideal of R containing both A and B.

14. If P is a prime ideal of commutative ring R then prove that R/P is a field.

15. Let 0 and 0' be zeros of the rings R and R' respectively. If f: R → R' is a homomorphism then prove that f(0) = 0' and f(-a) = - f(a)

16. Prove that every quotient group of a cyclic group is cyclic'

17. Prove that every field is an integral domain.

18. Show that a Boolean ring is commutative.

19. If A is an ideal of a ring R with unity such that 1 ∈ A then show that A = R.

20. Prove that an integral domain R with unity is a field iff R[x] is a PID.

21. If f: R → R' is a homomorphism then prove that Kerf is an ideal of R.

22. If f: R → R' is an onto homomorphism where R is a ring with unity then show that f(1) is unity of R'.

23. Let R be a commutative ring with unity Show that every maximal ideal of R is prime.

24. Prove that a commutative ring R is an integral domain iff cancellation law holds.

25. Prove that Every Boolean ring is of the order 2^n

26. A non-empty subset S of ring R is a subring of R iff a,b S ⟹ab∈S, a-b ∈S.

27. If F is a field then prove that F[x] is a Euclidean domain.

28. If F is a field then prove that every ideal in f[x] is principal.

29. For any commutative ring R with unity then prove that R[x]<x>≅R.

30.Let R be a commutative ring with unity such that R[x] is a PID, then prove that R is a field.