

Shivaji University, Kolhapur

Question Bank for Mar 2022 (Summer) Examination

Sub Code : 79698 Subject Name : 79698 Statistics Paper IX

Probability Distributions

Q.1 Choose the most correct alternative.

8) If $(X, Y) \sim BN(1, 2, 4^2, 5^2, 0.5)$ then the conditional distribution of X given $Y = 2$ is

- i) $N(2, 7)$
- ii) $N(1, 12)$
- iii) $N(1, 6)$
- iv) None of these.

9) Median of the Laplace distribution (dist^n) with parameters (μ, λ) is

- i) λ
- ii) μ
- iii) $\mu + \lambda$
- iv) $\mu - \lambda$

10) If X follows Laplace distribution with parameter (μ, λ) then its third central moment is

- i) λ
- ii) $\mu + \lambda$
- iii) 0
- iv) None of these.

11) If X follows Lognormal distribution with parameters $\mu = 6, \sigma^2 = 5$ then mode of X is

- i) 4
- ii) $\frac{1}{e}$
- iii) 5
- iv) e

12) If X follows Lognormal distribution with parameters (μ, σ^2) then $E(\log X)$ is

- i) μ
- ii) $\frac{1}{\mu}$
- iii) μ^2
- iv) None of these.

13) Which of the following distributions can be expressed in terms of power series

- i) Binomial
- ii) Pareto
- iii) Logistic
- iv) None of these.

14) Following is the p.m.f. of truncated Poisson distribution, truncated to the at $X = 0$ $P(X = k) = C \frac{e^{-\lambda} \lambda^k}{k!}, k = 1, 2, \dots$ then the value of C is

- i) $\frac{1}{\lambda}$
- ii) $(1 - e^{-\lambda})^{-1}$
- iii) $(1 - e^{-\lambda})$
- iv) λ

15) If $X \sim N(0, 1)$ truncated to the right at above $b=5$ then the p.d.f. of X is

$$f(X) = K e^{-\frac{x^2}{2}}, X < 5 \text{ then } K = \underline{\hspace{2cm}}.$$

- i) $\frac{1}{\sqrt{2\pi F(5)}}$
- ii) $\frac{1}{\sqrt{2\pi(1-F(5))}}$
- iii) $\frac{1}{F(5)}$
- iv) $\frac{1}{\sqrt{2\pi}}$

16) If $(X, Y) \sim BN(0, 0, 1, 1, \rho)$ then distribution of $\frac{X}{Y}$ is

- i) Normal
- ii) Bivariate normal
- iii) Cauchy
- iv) Uniform

17) If X follows Weibull distribution with parameter (α, β) then X^β follows

- i) Exponential
- ii) Lognormal
- iii) Cauchy
- iv) Normal

18) If X follows Logistic distribution with parameters $(5, 2)$ then mean and mode of X is

- i) 5, 5
- ii) 5, 2
- iii) 2, 5
- iv) 2, 2

19) If X follows truncated Normal distribution, truncated to the left below at a and to the right above at b then

- i) $P(X > a) = 0$
- ii) $P(X < a) = 0, P(X > b) = 0$
- iii) $P(X < b) = 0$
- iv) $P(a < X < b) = 0$

20) If X is truncated Exponential variate with parameter Θ , truncated to the left below at a then $E(X)$ is

- i) $\frac{1}{\Theta} + a$
- ii) $\frac{1}{\Theta^2}$
- iii) $\frac{1}{\Theta}$
- iv) a

21) If X follows Cauchy distribution with parameters (μ, λ) then Q.D. is

- i) $\mu - \lambda$
- ii) $\mu + \lambda$
- iii) μ
- iv) λ

22) If X follows Pareto distribution with parameters $\alpha = 3$, $\beta = 5$ Then $E(X)$ is

- i) 7.5
- ii) 8.5
- iii) 7
- iv) 8

23) The first ordered raw moment μ_1 about origin of a discrete random variable which follows truncated Poisson distribution with parameter m truncated at $X = 0$ is

- i) $\frac{m^2}{1-e^{-m}}$
- ii) $\frac{2}{1-e^{-m}}$
- iii) $\frac{m}{1-e^{-m}}$
- iv) None of these.

24) If $(X, Y) \sim BN(1, 2, 9, 16, 0.5)$ then $P(X \geq 3)$ is [given $\phi(0.67) = 0.74857$]

- i) 0.74537
- ii) 0.25143
- iii) 0.24537
- iv) 0.75463

25) If X follows Lognormal distribution with parameters $\mu = 3, \sigma^2 = 4$ then $E(X)$ is

- i) e^3
- ii) e^4
- iii) e^7
- iv) None of these.

26) If X follows Weibull distribution with parameters $\alpha = 1, \beta = 1$ then c.d.f. of x is

- i) e^{-x}
- ii) $1 + e^{-x}$
- iii) $1 - e^{-x}$
- iv) $1 - e^x$

27) If x follows Pareto distribution with parameters $(\alpha > 2, \beta)$ then $v(x)$ is

- i) $\frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}$
- ii) $\frac{\alpha\beta^2}{(\alpha-1)^2}$
- iii) $\frac{\alpha\beta^2}{(\alpha-2)}$
- iv) None of these.

28) The mean of truncated Binomial distribution, truncated left at $x=0$ with parameters (n, p) is

- i) np
- ii) $\frac{np}{1-q^n}$
- iii) $\frac{nq}{(1-q)^n}$
- iv) None of these.

29) The probability distribution for which independence and uncorrelatedness imply each other is

- i) Bivariate Normal
- ii) Exponential
- iii) Laplace
- iv) None of these.

30) If X is a random variable having p.m.f.

$$P(X) = C \cdot {}^{10}C_x \left(\frac{1}{2}\right)^X \left(\frac{1}{2}\right)^{10-X}, X = 1, 2, \dots, 10 \text{ then the value of } C \text{ is}$$

- i) $\frac{1}{\left[1 - \left(\frac{1}{2}\right)^{10}\right]}$
- ii) $\left[1 - \left(\frac{1}{2}\right)^{10}\right]$
- iii) $\left(\frac{1}{2}\right)^{10}$
- iv) $\left[1 + \left(\frac{1}{2}\right)^{10}\right]$

31) The probability curve for Lognormal distribution is _____.

- i) Mesokurtic
- ii) Platykurtic
- iii) Leptokurtic
- iv) None of these.

32) If x_1, x_2, \dots, x_k follows Multinomial distribution with parameters n, P_1, P_2, \dots, P_k then marginal distribution of x_1 is

- i) Poisson
- ii) Binomial
- iii) Hypergeometric
- iv) Uniform

33) If $(x, y) \sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then $E(Y/X = x)$ is

- i) μ_1
- ii) μ_2
- iii) $\mu_2 + \rho \frac{\sigma_2}{\sigma_1} (x - \mu_1)$
- iv) $\mu_2 + \rho \frac{\sigma_1}{\sigma_2} (x - \mu_1)$

34) If $(x, y) \sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then $V(Y/X = x)$ is

- i) σ_1^2
- ii) σ_2^2
- iii) $\sigma_2^2(1 - \rho^2)$
- iv) $\sigma_1^2(1 - \rho^2)$

35) If $(x, y) \sim BN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then $U = \left(\frac{x-\mu_1}{\sigma_1} + \frac{y-\mu_2}{\sigma_2} \right)$ follows

- i) $N(0, 1+\rho)$
- ii) $N(0, 1-\rho)$
- iii) $N(0, 2(1+\rho))$
- iv) $N(0, 2(1-\rho))$

36) If x follows Lognormal distribution with parameters (μ, σ^2) then mean of x is

- i) e^{σ^2}
- ii) $e^{\mu + \frac{\sigma^2}{2}}$
- iii) $e^{\sigma^2 + \frac{\mu}{2}}$
- iv) e^μ

37) If x follows Cauchy distribution with parameters (μ, λ) then coefficient of Q.D. is

- i) $\frac{\lambda}{\mu}$
- ii) $\frac{\lambda}{\mu+\lambda}$
- iii) $\frac{\mu}{\mu+\lambda}$
- iv) None of these.

38) If x follows Logistic distribution with parameters (μ, λ) then the mode of x is

- i) σ
- ii) μ
- iii) $\mu+\sigma$
- iv) None of these.

39) Following is the p.m.f. of truncated Poisson distribution, truncated at $x = 0$ $P(x) = c \frac{e^{-m} m^x}{x!}$, $x = 1, 2, \dots$ then the value of C is

- i) m
- ii) $\frac{1}{m}$
- iii) $1 - e^{-m}$
- iv) $\frac{1}{1-e^{-m}}$

40) If $X \sim LN(5, 4)$ then $E(X)$ is

- i) e^6
- ii) e^7
- iii) e^5
- iv) e^4

41) The median of a Lognormal distribution with parameters μ and σ^2 is

- i) e^μ
- ii) e^{σ^2}
- iii) $e^{\mu + \sigma^2}$
- iv) None of these.

42) If $X \sim LN(25, 5)$ then $V(\log X)$ is

- | | |
|---------------|----------------|
| i) 25 | ii) 5 |
| iii) e^{30} | iv) $e^{52.5}$ |

43) If X follows truncated Exponential distribution with parameters $\Theta=4$, truncated to the left below 2 then mean and variance of X are

- | | |
|----------------------------------|---------------------------------|
| i) $\frac{4}{9}, \frac{1}{16}$ | ii) $\frac{9}{4}, \frac{1}{16}$ |
| iii) $\frac{1}{16}, \frac{9}{4}$ | iv) None of these. |

44) Random variable X follows t-distribution with n.d.f. If $n=1$ then distribution of X is

- | | |
|-------------|-----------------|
| i) Normal | ii) Exponential |
| iii) Cauchy | iv) Weibull |

45) If X_1, X_2, X_3, X_4 are independent $N(0, 1)$ variates then $\frac{X_1}{X_2} + \frac{X_3}{X_4}$ follows

- | | |
|----------------|---------------|
| i) $C(0, 2)$ | ii) $C(0, 1)$ |
| iii) $C(0, 4)$ | iv) $C(2, 2)$ |

46) If $X_1, X_2, X_3, \dots, X_n$ are i.i.d. Cauchy random variates with parameters (μ, λ) then the distribution of \bar{X}_n is

- | | |
|----------------|-----------------------|
| i) $C(n, n^2)$ | ii) $C(n, n)$ |
| iii) $C(0, 1)$ | iv) $C(\mu, \lambda)$ |

47) If $(X, Y) \sim BN(2, 4, 1, 9, 0.5)$ then the distribution of $X-Y$ is

- | | |
|----------------|----------------|
| i) $N(-2, 7)$ | ii) $N(-2, 2)$ |
| iii) $N(0, 1)$ | iv) $N(7, 3)$ |

48) The mean of $B(2, 0.5)$ left truncated at $X=0$ is

- | | |
|------------------|--------------------|
| i) $\frac{3}{4}$ | ii) $\frac{4}{3}$ |
| iii) 4 | iv) None of these. |

Q.2 Attempt any 02 questions**8 Marks**

- 1) Obtain distⁿ function of Cauchy distⁿ and hence find quartiles.
- 2) Define truncated Binomial distⁿ, truncated to the left at X = 0 and obtain its mean & variance.
- 3) Obtain the distribution function of Laplace distⁿ and hence find quartiles.
- 4) Define Multinomial distⁿ and obtain means and variances.
- 5) Define truncated Poisson distⁿ, truncated to the left at X = 0 and obtain its mean & variance
- 6) Define Lognormal distribution with parameters (μ, σ^2). Find its mean and variance.
- 7) If $(X,Y) \sim BN ((\mu_1, \mu_1, \sigma^2, \sigma^2, p))$ then obtain the distribution $aX+bY+c$, where a,b and c are real numbers.
- 8) Obtain distⁿ function of Cauchy distⁿ with parameters (μ, λ) and hence find quartile deviation.
- 9) If $(X,Y) \sim BN ((0,0, \sigma^2, \sigma^2, p))$ then obtain the distribution X/Y
- 10) Obtain first and second raw moments about origin of Lognormal distribution with parameters (μ, σ^2).
- 11) Define power series distribution. Show that Binomial, Poisson and Geometric distribution are particular cases of power series distribution.
- 12) If $(X,Y) \sim BN ((0,0, \sigma^2, \sigma^2, p))$ then obtain m.g.f. of (X,Y) . Also find $E(X)$ and $E(Y)$.

- 13) Define Pareto distribution and find its mean and variance.
- 14) Obtain mean of truncated normal distribution with parameters (μ, σ^2) , truncated to the right above b.
- 15) Define Laplace distribution with parameters (μ, λ) . Find its m.g.f. and hence mean.
- 16) If $(X, Y) \sim BN ((0, 0, \sigma^2, \sigma^2, p))$ then find the distribution of $Z = X/Y$.
- 17) If $(X, Y) \sim BN ((\mu_1, \mu_1, \sigma^2, \sigma^2, p))$ then find
- i) Marginal distribution of Y
 - ii) Conditional distribution of Y given $X = x$

Q.3. Attempt any 04 questions 4 Marks

- 1) Obtain distribution function of Weibull distribution.
- 2) Define Logistic distribution with parameters (μ, σ) and find its mean.
- 3) Define Multinomial distribution and obtain its m.g.f.
- 4) If X follows Pareto distribution with parameters (1,2) then find $P(X < 3)$.
- 5) Obtain mean and variance of truncated Exponential distribution, truncated to the left below a.
- 6) Prove that independence & uncorrelatedness imply each other of BND.
- 7) Obtain median of Lognormal distribution.

- 8) Obtain mean of truncated Binomial distribution, truncated to the left at $X = 0$.
- 9) If $(X, Y) \sim BN ((\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, p))$ then obtain marginal distribution of X .
- 10) If $(X, Y) \sim BN ((0, 0, 1, 1, p))$ then find the correlation coefficient between X^2 and Y^2 .
- 11) If X follows Laplace distribution with parameters $(\mu=0, \lambda=1)$ then find $P(1 \leq |X| \leq 2)$.
- 12) Find mean of Weibull distribution with parameters (α, β) .
- 13) Define Logistic distribution with parameters (μ, σ) and find its mode.
- 14) If $(X, Y) \sim BN ((\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, p))$ then find conditional distribution of X given $Y = y$.
- 15) Define power series distribution and show that binomial distribution is a particular case of power series distribution.
- 16) Obtain mean of truncated normal distribution, truncated to the right above b , with parameters (μ, σ^2)
- 17) If X follows Cauchy distribution with parameters $(\mu=0, \lambda=1)$ then find the distribution of X^2 .

18) If $(X, Y) \sim BN(1, 2, 4^2, 5^2, 12/13)$ then find

i) $P(X > 2)$

ii) $P(X > 2 | Y = c)$

Given that $\Phi(0.25) = 0.59871$ and $\Phi(0.65) = 0.74215$ where Φ is the cdf of SNV.

19) A fair die is rolled 6 times. Find the probability that number 1 appears 2 times, number 2 or 3 appears 2 times, number 4 or 5 appears 1 time and number 6 appears one time.

20) Obtain the distribution function of the random variable X which follows Laplace distribution with parameters (μ, λ) .

21) State Cumulative distribution function of Cauchy distribution. Hence obtain quartiles.

22) Define truncated exponential distribution, truncated to the left below at $X = A$ also find its mean.

23) Obtain m.g.f. of Laplace distribution.

24) Define Logistic distribution obtain its distribution function.

25) Obtain correlation coefficient for multinomial distribution.

26) In trinomial case obtain its Marginal distribution.

27) State and prove additive property of multinomial distribution.

28) With usual notations obtain mean and variance of Lognormal distribution.

29) Obtain mode of Weibull distribution.

30) A fair die is rolled 12 times, find probability that

- i) 1 will occur twice
- ii) 3 will occur 3 times
- iii) 5 will occur once
- iv) 6 will occur twice