Shivaji University , Kolhapur
Question Bank For Mar 2022 (Summer) Examination

Subject Code : 81692 Subject Name : Probability Theory and applications

1. If X_1, X2, X3 is random sample from the p.d.f. f(x)=2X, if 0<x< 1 then pdf of third order statistic (Y) where 0<Y<1 is ………..

a) $24Y^5(1-Y^2)$ b) $Y^5(1-Y^2)$

c) $24(1-Y^2)$ d) None of these

2.If $P(X_n=0)=1-(1/n)$, $P(X_n=1)=(1/n)$,n=1,2,……… then

a) $X_n^2 \to 1$ b) $X_n^2 \to 2$

c) $X_n^2 \to 0$ c) None of these

3.If $X_n \xrightarrow{p} X$ then…

a) $X_n^2 \xrightarrow{p} X$ b) $KX_n^2 \xrightarrow{p} KX$

c) $X_n^2 - X \xrightarrow{p} 0$ d) $X_n^2 \xrightarrow{p} X^2$

4.If Y follows B(n,p) then proportion of success to the no.of trials (Y_n/n) converges to …… in probability as $n\to\infty$

a) P b) Q

c) P/n d) None of these

5. If $X_n \xrightarrow{p}$ a where a>0 then ………

a) $X_n^2 \to$ a b)$\sqrt{Xn}\, p \to$ a

c) $\frac{1}{X_n} \xrightarrow{p} \frac{1}{a}$ d) a$X_n \xrightarrow{p} a$

6. Convergence in probability of sample mean to population mean is implied by…..

a) CLT b) WLLN

c) Convergence in Q.M. d) All of the above

7. Sequence of r.v.'s {Xn , n≥1} is said to converge to X in distribution function if …..

a) $Fn(x) = 1$ b) $F(x) = 0$

c) $Fn(x) < 1$ d) None of these

8.If X1, X2, X3,…….Xn is random sample from the p.d.f. f(x) and c.d.f. F(x) then pdf of nth order statistic (Y) is ………..

a) nF(y)n-1f(y) b) nf(y)n-1F(y)

c) nF(y)nf(y) d) n[1-F(y)n-1]f(y)

9.If X1, X2, X3,…….Xn is random sample from the U(0,1) then rth order statistic is ………..

a) β1(r-1,n-r-1) b) β1(r,n-r+1)

c) β2(r-1,n-r-1) d) β2(r-1,n-r-1)

10.If Xn p → X then…

a) Xn_2 p → X b) KXn_2 p → KX

c) (Xn –X) p → 0 d) Xn^2 p → X^2

11. Convergence in probability of sample mean to population mean is known as ----

a) Weak law of large number b) Central limit theorem

c) Both a and b d) None of these

12. $\frac{X_n}{Y_n} \rightarrow \frac{X}{Y}$ is possible only when

a) P[Yn= 0] = P[Y=0]=1 b) P[Yn=0] = P[Y=0]=0

c) P[Xn=0]=P[X=0] =0 d) None of these

13.Reliability function of parallel system in terms of its component reliabilities P1 P2 P3 is given by

a)P1+ P2 +P3 – P1P2 – P2P3 – P1 P3 + P1P2P3

b) 1- P1P2P3

c)P1+P2+P3

d) None of these

14.For Geometric distribution with p.m.f p(x) = 2-x; x = 1,2,…the lower bound for P[IX-2I 2] is

a) ½ b) 15/16 c) ¾ d) 1

15.If Xn $p \to \ a$, where a>0 then

a) Xn2 $p \to \ a$ b)$\sqrt{Xn}$ $p \to \ a$

c)$\frac{1}{xn}$ $p \to \frac{1}{a}$ d) aXn $p \to \ a$

16. A state vector X is called Cut vector if value of structure function is …...

A) 0 B)1

C) Both A and B D) Neither A nor B

17. A binary system of n components is said to be coherent system if …...

a) Structure function is non decreasing function of vector X
b) All the components in system are relevant components
c) Both A and B
d) All the components in system are irrelevant components

18. A state vector X is called path vector if value of structure function is …...

a) 0 b) 1

c) Both a and b d) Neither A nor B

19.If X1, X2, …. Is a sequence of i.i.d. r.v. following Poisson distribution with parameter λ, then $\sum_{i=1}^{n} X_i$ follows -------

a) Poisson distribution b) Normal distribution

c) Asymptotic normal distribution d) None of these

20. Structure function of a component of a system is;

a) Normal variable b) Gamma variable

c) Bernoulli variable d) Cauchy variable

21) Chebychev's inequality states

a) $P (|x-E(x)| \geq k\sigma) \geq 1/k^2$ b) $P(|x-E(x)| \geq k\sigma) < 1 /k^2$

c) $P(|x-E(x)| \leq k\sigma) \geq 1/k^2$ d) $P(|x-E(x)| \leq k\sigma) > 1/k^2$

22) If X~ Poisson (4) then by chebychev's inequlity we get P[X-4<4]≥ -----

a) .5 b) .1

c) .25 d) .75

23) If X~ B (4,1/2) then we get chebychev's inequality P[│X-2│≥2] ≤-------

a) .5 b) .1

c) .25 d) .75

24) If X_1, X_2, …..xn are i.i.d random variables drawn from a population with mean μ then W.L.L.N states……

a) $X_n \xrightarrow{p} \mu$ b) $\overline{Xn} \xrightarrow{p} \mu$

c) $\overline{Xn} \xrightarrow{p} \mu+1$ d) None of these

25) Let $\overline{X}$ be the mean of random sample of size 100 drawn from a distribution $x^2{}_{50}$ An approximate value of ------

P[49 < $\overline{X}$ <51] is ……..(given $\emptyset(-1) = 0.1581$) is

a) 0.6838 b) 0.3414

c) 0.3174 d) None of these

26) Chebyshev's inequality can be applicable for …... random variable.

a) Discrete b) Continuous

c) Both A and B d) Neither A nor B

27) Chebyshev's inequality is used to obtain ………...

a) Lower bound for probability b) Lower bound for variance

c) Upper bound for Variance d) Both B and C are true

28) The lower bound for Chebyshev's inequality with mean E(x), variance σ^2 and positive integer k is ……

a) $\frac{1}{k^2}$ b) $1 - \frac{1}{k^2}$

c) $\frac{1}{k}$ d) None of these

29) If a random variable X has E(X)=3 and E $(X^2) = 13$ then upper bound for P [|X-3|≥ 8] is ……

a) $\frac{1}{16}$ b) $\frac{1}{4}$

c) $\frac{15}{16}$ d) $\frac{3}{4}$

30) Structure function ϕ(X̲) of series system of n independent components is given by ....

a) ϕ(X̲) = Max { x_1, x_2 , ..., x_i ,....... x_n) b) ϕ(X̲) = $\prod_{i=1}^{n} X_i$

c) ϕ(X̲) = $\coprod_{i=1}^{n} X_i$ d) ϕ(X̲) = $\prod(1 - X_i)$

31) Structure function ϕ(X̲) of series system of n independent components is given by ....

a) $\phi(\underline{X})$ = Max { $x_{1,}\ x_2$, ..., x_i ,....... x_n) b)$\phi(\underline{X}) = \prod_{i=1}^{n} X_i$

c) $\phi(\underline{X})$ = Min { $x_{1,}\ x_2$, ..., x_i ,....... x_n) d) Both B and C

32) Structure function of series system of 2 independent components is……

a) X_1X_2 b) $1 – (1 - X_1X_2)$

c) $1-(1- X_1) (1- X_2)$ d) $X_1X_2 X_3$

33)Structure function of series system of 3 independent components is……

a) $1-(1- X_1) (1- X_2) (1- X_3)$ b) $X_1X_2 X_3$

c) $1 – (1 - X_1X_2 X_3)$ d) X_1X_2

34) The reliability function at t is……………function.

a) constant b) decreasing

c) increasing d) none of these

35) Which of the following statement is correct for survival function?

a) R(o)=1 b) R(t)=0 as t $\rightarrow \infty$

c)R(t) is non-increasing function d) All the above

36) Hazard rate h(t) is given by……

a) f(t)/R(t) b) R(t)/F(t) c)F'(t)/1-F(t) d)Both A & C

37) Which of the following distribution has constant Hazard rate?

a) Normal b) Gamma c) Exponential d) Uniform

38) The reliability of 2 out of 3 system with component reliabilities $P_1 = 0.4$, $P_2 = 0.6$, $P_3 = 0.5$ are…….

a)0.88 b)0.5

c)0.6 d) 0.12

39) The reliability of series system of 3 components with component reliabilities $P_1 = 0.6$, $P_2 = 0.45$, $P_3 = 0.65$ are…….

a)0.077 b) 0.923

c)0.1755 d) 0.0.602

40) The reliability of parallel system of 3 components with component reliabilities $P_1 = 0.6$, $P_2 = 0.45$, $P_3 = 0.65$ are…….

a)0.077 b) 0.923

c)0.175 d) 0.0.602

Q. 2) attempt any two questions

1) State and prove weak law of of large Numbers (W.L.L.N) for i.i.d. random variables.

2) Define a) Hazard rate

b) Hazard function

c) Survival function

d) IFR distribution

e) DFR distribution

3) Define orders Statistics for a r. s. of size b drawn from a continuous distribution f(x). For a r. s. of size n from U(a, b). Obtain the distribution of i) Minimum ii) Maximum , Order Statistics.

4) State and prove central Limit Theorem (C.L.T.) using m.g.f.

5) Define order Statistics for a r. s. of size n drawn from a continuous distribution f(X). State & Prove the p.d.f. of $Yi= X_{(i)}$, i^{th} order Statistics. Hence find the p.d.f. of smallest and largest order statistics.

6) State and prove the joint p.d.f. of i^{th} and j^{th} order Statistics [Yi & Yj]

7) State and Prove Chebychev's inequality for i) Discrete distribution and

ii) Continuous distributions

8) Define joint p.d.f of Yi & Yj . Let Y1 < Y2 < Y3 be an order Statistics of size 3 drawn from the uniform distribution (0,1). Find the distribution of sample range.

9) If T is a life time of a component having exponential distribution with parameter Θ , then find the

i) Distribution function of f(t)

ii) Survival function R(t)

iii) Hazard rate h(t)

iv) Verify $E(T) = \int_0^\infty R(t)\, dt$

10) Show that hazard rate of a series system of components having independent life time is summation of hazard rates of the components. Also life time of series system of independent components with independent IFR life times is IFR.

11) let $X_{(1)}$ be the 1st order statistics of random sample of size n drawn from the distribution having p.d.f.
If Z= n($X_{(1)}$ - then investigate the limiting distribution of Z.

12) Explain the term convergence in i) Probability ii) Distribution function iii) Quadratic mean let $X_1,X_2,\ldots,X_n$ be a sequence of i.i.d random variable with p.d.f. U(0,). Show that Ynas n , where Y_n = max $(X_1,X_2,\ldots,X_n)$

13) a) Let {X_n , n be a sequence of random variable with common distribution show that WLLN exist.

b) Let $X_1,X_2,\ldots,X_n$ be a random sample drawn from poisson distribution with parameter (1) then using CLT show that as n

14) Explain the following terms

i) Reliability of a component of system

ii) Reliability of a system

iii) Reliability of a series system

iv) Reliability of a parallel system

v) Reliability of 2 out of 3 system

15) Let $Y_1<Y_2<Y_3<Y_4<Y_5$ denote the order statistics of random sample of size 5 from exp(1) show that $Z_1=Y_2$ and $Z_2= Y_4-Y_2$ are independently distributed.

16) A continuous random variable X is show that E(X)= 7 $E(X^2)$= 53

i) what is the least value of P [3 < X < n]

ii) what is the greatest value of P [ ]

iii) what is the value of k that guarities P [ ]

17) Obtain the minimal path and cut representation of the following coherent system

i) The 2 out of 3 system

ii) The series system of n independent components

iii) The parallel system of n independent components

18) Let { $X_{(n)}$, n ≥ 1} be a sequence of i.i.d. r.v., each X_n has continuous distribution with density

$$f(x)_{xy} = \begin{cases} e^{-(Xn-\alpha)} & , \ Xn \geq \alpha \\ 0 & ; O.W. \end{cases}$$

Let Y_n = min ($X_1, X_2, \ldots, X_n$. Show that $Y_n \xrightarrow{P} \alpha$. as n → ∞.

19) Let $X_1, X_2, \ldots, X_n$ be i.i.d. r.v. having U(0,1). Find the distribution of $r = X_{(n)} - X_{(1)}$, where $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be order Statistics. Have find E(r) & Var(r).

20) If {X_n, n ≥ 1) be a sequence of r. v. defined by, $P[X_n = 0] = 1 - 1/n$ and $P[X_n = 1] = 1/n$

i) Test for convergence in quadratic mean to zero.

ii) Test for convergence in probability to zero.

Q. 3) attempt any four questions

1) Obtain distribution function of i^{th} order statistic

2) Show that WLLN holds good for a sequence of i.i.d random variables X_1, X_2, ….X_n having Poisson Distribution with parameter m.

3) Define

a) Convergence in probability

b) convergence in quadratic mean

4) Let X_1, X_2, ….X_n be a random sample from exponential distribution with mean $\frac{1}{\theta}$. Find the distribution of smallest and largest order statistic.

5) State and prove WLLN for i.i.d random variables with finite variance.

6) Let $X_{(n)}$ be n^{th} order statistic of random sample X_1, X_2, ….X_n from U(0, θ), $\theta > 0$ with distribution function Fn. Examine the Weak convergence of Fn to any distribution function F.

7) Obtain Minimal path vector and Minimal path set for series system. Also give its block diagram

8) Define Hazard rate, Hazard function and survival function

9) Define reliability of a system and obtain it for

a) Series system of two components

b) parallel system of two components

10) If $X \sim U(-\sqrt{3}, \sqrt{3})$ distribution then obtain upper bound for $P[|X-\mu| > \left(\frac{3}{2}\right)\sigma]$

11) Explain convergence in quadratic mean

12) Explain convergence in r^{th} mean

13) Let $X_{(1)}, X_{(2)}, X_{(3)}$ be the order statistic of random sample of size 3 from the U (0,1). Find the distribution of sample median

14) Obtain distribution of i^{th} order statistic

15) Show that a series system with two components is a coherent system.

16) Define path vector and minimal path vector.

17) State and prove WLLN.

18) Explain convergence in probability of sequence of random variables to

a) constant b) variable

19) A random variable has mean 12 and variance 9 find the bounds on $P[6 \leq X \leq 18]$

20) If X follows U (0,10) find the bound for $P[|X-5| > 4]$

21) Draw a reliability block diagram and give a structure function for 2 out of 4 system.

22) A system consists of seven identical components connected in parallel. What must be the reliability of each component if the overall reliability of the system is 0.9.

23) For a series system of two components having reliability 0.5 each. Find the reliability of the system.

24) Derive the reliability function for 2 out of 3 good system.

25) Derive the inter relation between survival function distribution function and hazard rate.

26) Let $\{X_k, k \geq 1\}$ be a sequence of independent random variable with

$$P(X_k = \pm 2^k) = \frac{1}{2^{2k+1}}$$

$$P(X_k = 0) = 1 - \frac{1}{2^{2k}}$$

Check whether WLLN holds or not.

27) Let $X_{(1)}$ be smallest order statistic corresponding to r. s. of size n drawn from the distribution having p.d.f. $f(x) = e^{-(x-\theta)}, x \geq 0, \theta > 0$ .Then show that $X_{(1)} \rightarrow \theta$ in probability as n $\rightarrow \infty$.

28) Let $X_{(1)}, X_{(2)}, X_{(3)}, X_{(4)}$ be order statistic corresponding to random sample of size 4 drawn from the distribution having pdf

$$f(x) = e^{-x} \quad x > o$$

Find $P(X_{(4)} \geq 3)$

29) Define reliability of a system and obtain it for 3 out of 4 system.

30) State different forms of Chebyshev's inequality.