SHIVAJI UNIVERSITY,KOLHAPUR

B.SC.II (SEM-III) MATHEMATICS

PAPER-V (Analysis-I)

Multiple choice Questions

1) If $A=\{x \in N \mid 2x-1\, is\ even\}$ and $B=\{x \in R \mid x^2-9=0\}$ then which of the following statements is true.
   a) A is empty and B is empty
   b) A is nonempty and B is empty
   c) A is nonempty and B is nonempty
   d) A is empty and B is nonempty
2) If $A=\phi$ and $B=\{\phi\}$ then which of the following statements is true.
   a) $\phi \in A$ b) $A \subset B$ c) $B \subset A$ d) $B=\phi$
3) If $f:R \to R$ is given by $f(x)=x^2+1$, then $f^{-1}(-5)$ is
   a) -5 b) 5 c) ϕ d) 0
4) If $f:R \to R$ is given by $f(x)=x^2$ then the range of f is
   a) R c) N
   b) R_1 (the set of nonnegative real numbers) d) Z
5) If $f:R^+ \to R$ (where R^+ is the set of positive real numbers) is given by $f(x)=\log x$ then the set $\{x \mid f(x)=-2\}$ is
   a) ϕ b) -2 c) e^2 d) e^{-2}
6) If $f:R \to R$ is given by $f(x)=|x|$ then f is
   a) Injection b) surjection c) bijection d) none of the above
7) Which of the following statements is correct?
   a) $a \in \{a,b,c\}$ b) $\{a\} \in \{a,b,c\}$ c) $a \subset \{a,b,c\}$ d) $\{\{a\},b,c\} \subseteq \{a,b,c\}$
8) If C and R be the sets of complex and real numbers respectively. If $f:C \to R$ is given by $f(z)=|z|$ then f is
   a) One-one b) onto c) one-one and onto d) neither one-one nor onto

9) The set $\{x \mid x < 7\}$ is the interval

a) (0, 7) b) $(-\infty, 7)$ c) [0, 7] d) $(-\infty, 7]$

10) If $A = \{1, 2, \{3\}, (4,5)\}$ then the number of elements in it is

a) 3 b) 4 c) 5 d) 2

11) If P is the set of prime integers, then which of the following is true

a) $7 \in P$ b) $11 \notin P$ c) $9 \in P$ d) $\{7\} \subset P$.

12) Set of real numbers is …

A) Uncountable
C) finite
B) Countable
D) none of these

13) If f is a function from A into B with range of $f = B$ then f is called …

A) Onto
C) one-one and onto
B) One-one
D) none of these

14) A function $f : A \to B$ is called a one-one correspondence between A and B, if …

A) f is neither one-one nor onto

B) f is one-one and onto

C) f is one-one but not onto

D) f is not one-one but onto

15) If $f : A \to B$ and if $f(a_1) = f(a_2)$ implies $a_1 = a_2$ for all $a_1, a_2 \in A$, then f is called … function.

A) Onto
C) one-one and onto
B) One-to- one
D) none of these

16) Set of all rational numbers in $(0,1)$ is …

A) Finite
C) countable
B) Uncountable
D) none of these

17) If f and g are two functions with respective domains X and Y then g is called extension of f onto Y if …

A) $X \supset Y$ and if $f[X] = g[X]$, for all $x \in X$

B) $X \subset Y$ and if $f[X] = g[X]$, for all $x \in X$

C) $X \subset Y$ and if $f[X] \neq g[X]$, for all $x \in X$

D) $X \supset Y$ and if $f[X] \neq g[X]$, for all $x \in X$

18) A function $f: A \to B$ is called a one-one correspondence between A and B, if …

A) f is one-one but not onto

B) f is one-one and onto

C) f is not one-one but onto

D) f is neither one-one nor onto

19) The set of … is uncountable set.

A) Positive integers

B) Integers

C) rational numbers

D) real numbers

20) If $f(x) = 1 + \sin x \ (-\infty < x < \infty)$ and $g(x) = x^2 \ (0 \le x < \infty)$ then $g \circ f(x) =$ …

A) $1 + \sin^2 x \quad (-\infty < x < \infty)$

B) $1 + 2\sin x + \sin^2 x \quad (0 \le x < \infty)$

C) $1 + \sin^2 x \quad (0 \le x < \infty)$

D) $1 + 2\sin x + \sin^2 x \quad (-\infty < x < \infty)$

21) If $f: A \to B, X \subset A, Y \subset A$, then $f(X \cap Y)$ is …

A) Equal to $f(X) \cup f(Y)$

B) Equal to $f(X) \cap f(Y)$

C) Not necessarily equal to $f(X) \cap f(Y)$

D) Not necessarily equal to $f(X) \cup f(Y)$

22) The set of rational numbers is …

A) Countable

B) Uncountable

C) finite

D) none of these

23) If $g(x) = x^2 (0 \le x < \infty)$, then $g^{-1}(x)....(0 \le x < \infty)$.

A) x^2 B) x C) $x^{3/2}$ D) $x^{1/2}$

24) If A is any non-empty subset of R that is bounded below, then A has … in R.

A) A greatest lower bound

B) A least upper bound

C) Upper bound

D) None of these

25) Let f be a real valued function described by $f(x) = x^2 (-\infty < x < \infty)$. Then $f([0,3)) =$ …

A) (0, 9) B) (0, 9] C) [0, 9) D) [0, 9]

26) The validity of statement $p(n)$ is proved by using mathematical induction for even

a) Real number b) integer c) natural number d) rational number

27) Well ordering principle states that every nonempty subset of natural number N has a ….. element.

a) One b) infinite c) no d) least

28) The condition p(1) is true is ………. Condition for proving validity of p(n) by mathematical induction.

a) Necessary b) sufficient
b) Necessary and sufficient d) neither necessary nor sufficient

29) If p(n) is true for $n = n_0$ and p(k) is true implies p(k+1) is true, then this type of mathematical induction is called ……. Version of mathematical induction.

a) First b) second c) regular d) principal

30) By mathematical induction, the result $2^n < n!$ is true for all…..

a) $n \geq 2$ b) $n \geq 3$ c) $n \geq 4$ d) $n \geq 1$

31) The result $n^2 < 2^n$ for all $n \in N$ is not true for all $n \in N$.

a) Not true for n=1 c) both a) and b)
b) Truth for n=k does not imply truth for n=k+1 d) none of these

32) By using second version of mathematical induction the result

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{7}{\sqrt{n}} > \sqrt{n} \text{ is true for all ….}$$

a) $n \geq 1$ b) $n \geq 2$ c) $n \geq 3$ d) $n \geq 4$

33) The result for every subset S of N if $1 \in S$ and for every $K \in N$, $\{1,2,3,.....\} \leq S$ then S=N is called …………

a) Principle of Mathematical induction
b) First version of Mathematical induction
c) second version of Mathematical induction
d) principle of strong induction

34) The result for every nonempty set S of N if $1 \in S$ and for every $K \in N, K \in S$ then $K + 1 \in S$ then S=N is called ………..

a) Principle of Mathematical induction
b) First version of Mathematical induction
c) second version of Mathematical induction
d) principle of strong induction

35) The result n=n+2 is false by mathematical induction because ………….

a) It is not true for n=1
b) Truth for n=k does not imply truth for n=k+1
c) both a) and b)
d) none of these

36) Every 1-1 correspondence is ………..

a) One-one b) onto c) countable d) all a), b), c).

37) The function $f(x)=x^2, x\in R$ is …………..

a) One-one b) onto
c) neither one-one nor onto d) none of these

38) The set of rational numbers is …………..

a) Finite b) countable c) uncountable d) none of these

39) The set of natural numbers is …………

a) Finite b) countable c) uncountable d) none of these

40) The set of integers is …………

a) Finite b) countable c) uncountable d) none of these

41) The set of real numbers is ……….

a) Finite b) countable c) uncountable d) none of these

42) The closed interval [0, 1] is …………

a) Countable
b) Uncountable
c) neither countable nor uncountable
d) finite

43) The open interval (0, 1) is …………

a) Countable
b) Uncountable
c) neither countable nor uncountable
d) finite

44) The set of rational numbers in [0, 1] is …………

a) Countable
b) Uncountable
c) neither countable nor uncountable
d) finite

45) The Cartesian product $Z\times Z$ where Z is the set of integers is …………..

a) Countable
b) Uncountable
c) neither countable nor uncountable
d) finite

46) The Cartesian product of two countable sets is ……….

a) Countable   c) neither countable nor uncountable

b) Uncountable   d) finite

47) The function $f(x)=\cos x$ in [0, 1] is ……

a) One-one   b) onto   c) one-one and onto   d) none of these

48) The union of countable sets is …………..

a) Countable   c) neither countable nor uncountable

b) Uncountable   d) none of these

49) If the set A is equivalent with set of integers then the set A is ……..

a) Finite   b) countable   c) uncountable   d) none of these

50) The set of all ordered pairs of integers is ……………

a) Finite   b) countable   c) uncountable   d) none of these

10 MARKS QUESTIONS :

1) Define direct and inverse image of a subset. If $f: A \to B$ and if $X \subseteq B, Y \subseteq$ B then show that $f^{-1}(XUY) = f^{-1}(X) \cup f^{-1}(Y)$.
2) Define direct and inverse image of a subset. If $f: A \to B$ and if $X \subseteq B, Y \subseteq$ B then show that $f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$.
3) Define inverse function. If $f: A \to B$ and g: $B \to C$ are functions and let H be a subset of C. Then show that $(g \circ f)^{-1}(H) = f^{-1}\left(g^{-1}(H)\right)$.
4) Define Injective, Bijective and Surjective functions. If $f: A \to B$ is injective and $E \subseteq A$ then show that $f^{-1}\left(f(E)\right) = E$.
5) State Principal of Mathematical Induction. By using mathematical induction, prove that $1 + 2 + 3 \dots \dots \dots \dots + n = \frac{n(n+1)}{2}$.
6) State Principal of Mathematical Induction. By using mathematical induction, prove that $1^2 + 2^2 + 3^2 + \cdots \dots \dots + (2n-1)^2 = \frac{n(4n^2-1)}{3}$.

7) State Principal of Mathematical Induction. By using mathematical induction, prove that

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\ldots\ldots+\frac{1}{n.(n+)}=\frac{n}{n+1}.$$

8) If A_m is a countable set for each $m \in N$, then show that the union $A = \bigcup_{m=1}^{\infty} A_m$ is countable.

9) Prove that the set of all rational numbers is countable.

10) Prove that the closed interval $[0, 1]$ is uncountable.

11) Prove that i) Union of two disjoint countable sets is countable.
ii) Any subset of a countable set is countable.

12) State and prove Arithmetic- Geometric Mean enequlity.

13) If $a \in R$ and $a \neq 0$ then prove that
i) $a^2 > 0$ ii) $1 > 0$ iii) If $n \in N$, then prove that $n > 0$.

14) If $x > 1$, then prove that Bernoulli's inequality $(1 + x)^n \geq 1 + nx$ for all $n \in N$.

15) If $a, b \in R$, then prove that i) $|a + b| \leq |a| + |b|$,
ii) $\big||a| - |b|\big| \leq |a - b|$.

Questions for 5 marks

1. If A and B are any two sets then prove that $(A \cup B)' = A' \cap B'$.
2. If A and B are any two sets then prove that $A - B = A \cap B'$.
3. If $A = \{x/x^2 - 8x + 15 = 0\}$, $B = \{x/x^2 - 7x + 10 = 0\}$ and $C = \{x/x^2 - 4x + 3 = 0\}$, then write the sets$i)\ A \cup B,\ \ ii)\ A \cap B,\ \ iii) B \cup C,\ \ iv)\ A \cap C,\ \ v)\ A \cup (B \cap C)$.
4. A relation R on the set $\{0,1,2,3, \ldots\ldots\ldots 10\}$ defined by the equation$2x + 3y = 12$, then write the relation as the set of ordered pairs.
5. Determine the range and domain of the relation R defined by $R = \{(x, y)/x\ is\ a\ prime\ number\ less\ than\ 20\ and\ y = x^3\}$.
6. If $f; R \to R$ be a function defined by $f(x) = 3x + 7$, then show that the function f is one-one and onto. Also find f^{-1}.
7. If $f; N \to N$ is defined by $f(x) = \begin{cases} \frac{n+1}{2} & if\ n\ is\ odd \\ \frac{n}{2} & if\ n\ even \end{cases}$
then show that f is not one-one but it is onto function.

8. Show that the function $f; N \to N$ given by $f(n) = n - (-1)^n$ is a injective function for all n.
9. Prove that $5^{2n} - 1$ is divisible by 8 for all $n \in N$.
10. Prove that $3^{2n} + 7$ is divisible by 8 for all $n \in N$.
11. Prove that $n^3 + 5n$ is divisible by 6 for all $n \in N$.
12. Prove that for $n \in N$, $a^n - b^n$ is divisible by $a - b$ for all $n \in N$.
13. Prove that $2^n < n!$ for all $n \geq 4, n \in N$.
14. Prove that $n < 2^n$ for all $n \in N$.
15. Prove that the set of all real numbers is uncountable.
16. Prove that the set of integers is countable.
17. Prove that the set of natural numbers is countable.
18. Prove that the open interval (0, 1) is uncountable.
19. Show that the Cartesian product of two countable sets is also countable.
20. Prove that the sets of even and odd natural numbers are countable.
21. Prove that the set of all polynomial functions with integer coefficients is countable.
22. Find all values of x satisfying the inequality $|x - 3| > |x + 2|$.
23. Find all values of x satisfying $3x - 1 = |x - 7|$.
24. If $a \geq 0, b \geq 0$, then prove that $a < b \Leftrightarrow a^2 < b^2 \Leftrightarrow \sqrt{a} < \sqrt{b}$.
25. Determine the set A of all real numbers x such that $x^2 + x > 2$.
26. Find the real values of x, satisfying the inequality $2x + 1 \leq x + 5 \leq 3x + 4$.
27. Find the real values of x, satisfying the inequality $x^2 > 3x + 4$.
28. Determine the set of all real numbers x such that $|2x - 1| \leq x + 1$.
29. Find all values of $x \in R$ that satisfying the inequality $|x - 2| \leq |x + 1|$.
30. Find all values of $x \in R$ that satisfying the inequality $|x| + |x + 1| < 2$.

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B.SC.II (SEM-III) MATHEMATICS

PAPER-VI (ALGEBRA-I)

Multiple choice Questions

1) The Eigen values of the matrix $\begin{bmatrix} -1 & 0 & 0 \\ 1 & 3 & 0 \\ 2 & -2 & 4 \end{bmatrix}$ are …

A) -1, 1, 2 C) -1, 3, 4

B) 1, 2, -2 D) 3, -2, 4

2) The matrix $\begin{bmatrix} 1 & 3-i & 5+2i \\ 3+i & 1 & 6+2i \\ 5-2i & 6-2i & 0 \end{bmatrix}$ is …

A) Hermition C) Symmetric

B) Skew-Hermition D) Skew-Symmetric

3) The Eigen values of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ are …

A) 5, 3, 3 C) 5, -3, 3

B) 5, -3, -3 D) -5, 3, 3

4) The rank of matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 3 & 4 & 6 \end{bmatrix}$ is …

A) 1 C) 3

B) 2 D) 0

5) The rank of matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is …

A) 1 B) 3

C) 2 D) 0

6) The rank of matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ is …

A) 1 C) 2
B) 3 D) 0

7) In system of linear equations AX=0 if rank of A=number of variables then it has … solution.

A) Only trivial C) infinite
B) Non trivial D) no

8) The sum of eigen values of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ is …

A) 8 C) 9
B) 10 D) 0

9) The sum of eigen values of $\begin{bmatrix} 7 & 2 & 2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ is …

A) 4 C) 5
B) 6 D) 0

10) The sum of eigen values of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ is …

A) 4 C) 5
B) 6 D) 0

11) The product of eigen values of $\begin{bmatrix} 7 & 2 & 2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ is …

A) 21 C) -22
B) -21 D) 22

12) The product of eigen values of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ is …

A) 30 C) 29
B) 28 D) 27

13) The product of eigen values of $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$ is …

A) 4 C) 5
B) -4 D) -5

14) The characteristic equation of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ is …

A) $\lambda^3 + 20\lambda + 8 = 0$ C) $\lambda^3 - 20\lambda + 8 = 0$
B) $\lambda^3 - \lambda^2 + 20\lambda + 8 = 0$ D) $\lambda^3 + \lambda^2 + 20\lambda + 8 = 0$

15) The characteristic equation of $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ is …

A) $\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$ C) $\lambda^3 + 5\lambda^2 + 7\lambda - 3 = 0$
B) $\lambda^3 - 5\lambda^2 - 7\lambda - 3 = 0$ D) $\lambda^3 - 5\lambda^2 + 7\lambda + 3 = 0$

16) The characteristic equation of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is …

A) $\lambda^3 - 7\lambda - 36 = 0$ C) $\lambda^3 + 7\lambda - 36 = 0$
B) $\lambda^3 + 7\lambda + 36 = 0$ D) $\lambda^3 - 7\lambda + 36 = 0$

17) If two eigen values of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 15, then third eigen value is …

A) 0 B) 1 C) 2 D) 3

18) If one eigen value of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ is 6, then second eigen value is …

A) 0 B) 1 C) 2 D) 3

19) If two eigen values of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are -2 and 6, then third eigen value is …

A) 0 B) 1 C) 2 D) 3

20) Let A={1,2}, then A x A= -----------

a) {(1,1),(1,2),(2,2)}
b) {(1,1),(1,2),(2,1)}
c) {(1,1),(1,2),(2,1),(2,2)}
d) {(1,1),(2,2)}

21) If R is a relation on set $A = \{1,2,3,4,5,6,7,8,9,10\}$ defined as $(x, y) \in R$ if and only if 'x is square of y', then R is -------------

a) {(1,1),(4,16),(5,25)}
b) {(1,1),(2,2),(2,4)}
c) {(1,1),(4,2),(9,3)}
d) {(1,1),(3,9),(4,2)}

22) If set X contains m elements and Y contains n elements, then X x Y contains ------ elements.

a) $m - n$ b) $m + n$ c) mn d) $(mn)^2$

23) If $R = \{(a, b), (b, c), (a, c), (c, c)\}$ is the relation on a set $A = \{a, b, c\}$, then matrix of relation R is -----

a) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

24) If R={(1,2),(2,2),(1,3),(3,3)} is the relation on set A={1,2,3} then the matrix of relation R is ---------------

a) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

25) A relation R on set S which is reflexive, symmetric and transitive is called ------- relation.

a) Inverse b) partial order c) an equivalence d) anti-symmetric

26) Which of the following is true in Monoid G with binary operation $*$ …

A) Identity element is unique

B) Inverse element is unique

C) $(a*b)^{-1} = b^{-1} * a^{-1}, \forall a, b \in G$

D) All the above

27) In a group $G = \{1, -1, i, -i\}$, under multiplication the identity element is …

A) 1 B) -1 C) i D) –i

28) In a group $G = \{1, \omega, \omega^2\}$,under multiplication the identity element is …

A) **1** B) -1 C) ω D) ω^2

29) Which of the following is false in a group G?

A) a.b=b.a
B) Inverse element is unique
C)Identity element is unique
D) $(a*b)^{-1} = b^{-1} * a^{-1}, \forall a, b \in G$

30) The set $N = \{1,2,3,...\}$ of natural numbers under multiplication is a …

A) Trivial group　　　　C) Group

B) Monoid　　　　**D) Abelian group**

31) The set $I = \{...,-4,-3,-2,-1,0,1,2,3,...\}$ of all integers under addition is a …

C) Trivial group　　　　**C) Abelian group**

D) Monoid　　　　D) none of these

32) Let Q_+ be the set of all positive rational numbers and $*a$ binary operation on Q_+ defined by $a * b = \frac{ab}{3}$ then inverse of a is …

A) 3/a　　B) -3/a　　**C) 9/a**　　D) -9/a

33) Let R be set of all real numbers and $*a$ binary operation on R defined by $a * b = a + b + ab$ then identity element in R is …

A) $0; if \ a \neq 1$　　　　C) $1; if \ a \neq 0$

B) $0; if \ a \neq -1$　　　　D) $-1; if \ a \neq 1$

34) Let R be set of all real numbers and $*a$ binary operation on R defined by $a * b = a + b + ab$ then inverse of a in R is …

A) $-\frac{a}{a-1}$　　B) $\frac{a}{a-1}$　　**C)** $-\frac{a}{a+1}$　　D) $\frac{a}{a+1}$

35) The set $G = \{a + b\sqrt{2} : a, b \in Q\}$ is a …

A) Trivial group　　　　**C) Group**

B) Monoid　　　　D) Abelian group

36) With respect to addition of matrices the set of all $m \times n$ matrices having their elements as integers is …

A) Trivial group　　　　**C) an infinite abelian group**

B) Monoid　　　　D) an abelian group

37) With respect to matrix multiplication the set of all $n \times n$ non-singular matrices having their elements as rational numbers is …

A) Trivial group　　　　**C) an infinite non-abelian group**

B) Monoid　　　　D) an abelian group

38) Under matrix multiplication the set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, where α is a real number, forms a …

A) Trivial group　　　　**C) Group**

B) Monoid　　　　D) Abelian group

39) Under matrix multiplication the set

$G=\left\{\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \text{ is any non}-\text{zero real number}\right\}$, is a …

A) Trivial group
B) Monoid
C) Group
D) commutative group

40) Which of the following is not a group?

A) (Z, -)
B) (N, .)
C) (R, +)
D) (N, +)

41) Identity element of the group G under the operation $*$ given by $a*b=a+b-5$ is …

A) 1
B) 2
C) 5
D) -5

42) In a group $G=\{1,-1,i,-i\}$ the inverse of element i is …

A) 1
B) i
C) $-i$
D) -1

43) Which of the following structure is not a group?

A) (R, +)
B) $(Q^*,+)$
C) $(R^*,\bullet)$
D) $(I,\bullet)$

44) In a group $G=\{1,-1,i,-i\}$ the inverse of element $-i$ is …

A) 1
B) i
C) $-i$
D) -1

45) For any element $a\in G$, $\left(a^{-1}\right)^{-1}=$ …

A) E
B) a
C) 1
D) 0

46) A set which is closed under an associative binary operation is called a …

A) Group
B) **Semi-group**
C) subgroup
D) abelian group

47) If G is a group and $a\in G$, then the subset $\{x\in G \mid xa=ax\}$ is called …

A) **Normalizer of** $a\in G$
B) Center of G
C) digit coset of $a\in G$
D) none of these

48) For Euler's ϕ function, if n=8 then $\phi(n)=\ldots$

A) 10
B) 6
C) 4
D) 8

49) If G is a finite group and H is a subgroup of G then o(H)=…..

A) $\frac{o(H)}{o(G)}$
B) $o(G)$
C) $o(H)$
D) $\frac{o(G)}{o(H)}$

50) For Euler's ϕ function $\phi(10)=\ldots$

A) 10
B) 4
C) 5
D) none of these

Long Answer Questions (Algebra I)

1) A non-empty subset of H of a group G is a subgroup of G if and only if for all $a, b \in H \Rightarrow a * b^{-1} \in H$.
2) Define Normalizer of an element of a group. Prove that the Normalizer $N(a)$ of $a \in G$ is a subgroup of G .
3) Define Centre of a group G .Prove that the Centre $Z(G)$ is a subgroup of
4) Define cyclic group and if a is a generator of a cyclic group G, then show that $o(a) = o(G)$.
5) Prove that every subgroup of cyclic group is cyclic.
6) Define Euler Phi function. Hence prove that a cyclic group of order d has $\phi(d)$ generators.
7) If H is a subgroup of $G,$ then there exists one-to-one correspondence between any two right cosets of H in G .
8) State and prove Cayley Hamilton Theorem .
9) Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$
10) Verify Cayley Hamilton Theorem for the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

    Also find inverse of A .
11) Find the characteristic equation for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and use it to find the simplified expression for $A^5 + 5A^4 - 6A^3 + 2A^2 - 4A + 7I.$
12) Define the following terms with illustration
    i) Relation
    ii) Inverse relation
    iii) Reflexive relation
    iv) symmetric relation
    v) Transitive relation

13) Explain the steps involved in Warshalls algorithm. Hence if A={1, 2, 3, 4} and R={(1, 1), (1, 4), (2, 1), (2, 2), (2, 2), (3, 3), (4, 4)} then find the transitive closure of R using Warshall's algorithm.
14) Define equivalence relation and if R be an equivalence relation defined on A then prove that R induces a partition on A.
15) Define composition of relation. If A={2, 3, 4, 5} and the relations R and S on A defined by R={(2, 2), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5), (5, 3)} S={(2, 3), (2, 5), (3, 4), (3, 5), (4, 2), (4, 3), (4, 5), (5, 2), (5, 5)} then find
    i) Matrices of above ralations.
    ii) Use matrices to find the following compositions of the relation R and S
        a) $R \circ S$ b) $R \circ R$ c) $S \circ R$

Short Answer Questions (Algebra I)

1) Define the terms i) Algebraic structure ii) Groupoid iii) Semi geoup
2) Define the terms i) Monoid ii) Group
3) Let $(G,*)$ be a group then prove that the identity element e is unique.
4) Let $(G,*)$ be a group then prove that inverse of each element in G is unique.
5) Let $(G,*)$ be a group then prove that $\left(a^{-1}\right)^{-1} = a$ for all $a \in G$.
6) Let $(G,*)$ be a group then prove that $(a*b)^{-1} = b^{-1} * a^{-1}; \forall a,b \in G.$
7) If G is a group with binary operation $*$ and if a and b are any elements of G, then linear equation $a*x=b$ has a solution in $G.$
8) Show that the set I of all integers $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$ is an abelian group with respect to the operation of addition of integers.
9) Show that the set $G=\{\ldots,-3m,-2m,-m,0,m,2m,3m,\ldots\}$ of multiples of integers by a fixed integer $'m'$ is a group with respect to addition.
10) Show that the set of all positive rational numbers forms an abelian group under the composition defined by, $a*b=\frac{(ab)}{2}$

11) Show that the set $\{1,-1,i,-i\}$ is an abelian group of order 4 under multiplication.

12) Show that the set $G=\{a+b\sqrt{2} \mid a,b\in Q\}$ is a group with respect to addition.

13) Show that the set I of all integers is an abelian group with operation defined by $a*b=a+b+1$ for all $a,b\in I$.

14) How many generators are there of the cyclic group of order 8.

15) Show that the group $\{1,-1,i,-i\}$ is cyclic with respect to multiplication.

16) Define Hermitian and Skew Hermitian matrix .

17) Show that every square matrix can be uniquely expressed as the sum of Hermitian and Skew Hermitian matrix.

18) If A is Hermitian matrix then prove that iA is Skew Hermitian matrix .

19) If A is Skew Hermitian matrix then prove that iA is Hermitian matrix .

20) Prove that the matrix $A=\begin{bmatrix} 3 & 5+2i & -3 \\ 5-2i & 7 & 4i \\ -3 & -4i & 5 \end{bmatrix}$ is Hermitian matrix .

21) Reduce the matrix $A=\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ to echelon form and find its rank .

22) Find characteristic equation of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

23) Define Reflexive , Symmetric and Equivalence relation .

24) Solve homogeneous linear equations $x-2y+3z=0,\ 2x+5y+6z=0$.

25) Solve non homogeneous equations

$$x+y+z=6,\ x+2y+3z=14,\ x+4y+7z=30$$

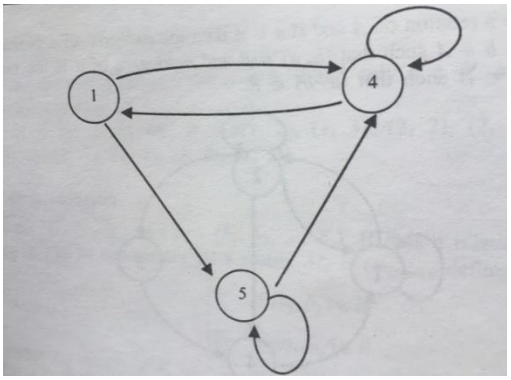
26) Using Gauss elimination method to solve the equations

$$2x+y+z=10;\ 3x+2y+3z=18;\ x+4y+9z=16$$

27) Prove that the relation ‘congruence modulo n’ on set of integers is an equivalence relation .

28) Let $A=\{1,2,3,4,6\}$ and R be a relation on A such that $(a,b)\epsilon R$ if and only if $2a=b$. Find the domain, range, matrix and the diagraph of R .

29) For the diagraph shown in the following figure, find R and M_R.



30) If $A = \{1,2,3,4\}$ and AxA is an equivalence relation on A then find $^{A}/_{R}$