Yashwantrao Chavan College of Science, Karad
Department of Computer Science Question Bank,2023-2024
Subject: Mathematics
Paper No-II
GEC-106 Algebra

1. Find DNF of Boolean expression E $((x_1, x_2) = (x_1 \vee x_2) \vee (x_1 \vee x_2)$
2. If Z is the set of all integers and '*' is a binary operation defined on Z by a*b = a+b-2,∀ a,b∈ Z then show that $< Z,* >$ is group.
3. The Relation R is defined on the set Z as xRy⇔ $x^2 = y^2$.Show that R is an equivalence relation.
4. Define
   1] Group
   2] Subgroup of a group G
5. Find G.C.D of 3587 and 1819 and express it in the form m(3587) + n(1819).
6. Find G.C.D of 7469 and 2464 and express it in the form m(7469) + n(2464).
7. Find G.C.D of 4999 and 1109 and express it in the form m(4999) + n(1109).
8. If (G, *) is a group then prove that, Identity element in G is unique.
9. If c/ab and (b,c)=1 then c/a.
10. If a/b and a/c then prove that a|(ax+cy).
11. If (G,*) is a group then prove that for all a,bϵG, $(a * b)^{-1} = (b * a)^{-1}$.
12. If $A_1 = \{1,2,3,4\}, A_2 = \{4,5,6\}$ *and* $A_1 = \{6,7,8\}$ Show that
   1] $A_1 \cup A_2$
   2] $A_2 \cup A_3$
13. In a lattice $(L, \leq)$ for a,b,c ∈ L prove that,
   1] $a \vee (a \wedge b) = a$
   2] $a \wedge (a \vee b) = a$
14. Let R be equivalence relation then show that any two equivalence classes are either disjoint or identical.

15. Find the remainder when $4^{37} + 82$ is divided by 7.

16. Let A={$(x, y/1,2,3)$}, B={3,4} be a relation on A defined as $(x, y) \in R$ iff
   1] x is divisible by y
   2] y is divisible by x   Find a Graph of R.

17. Given an example of a relation which is Transitive but Neither Reflexive nor Symmetric, iff $x < y$.

18. Show that $G = \{1, -1, i, -i\}$is a group under multiplication where $i^2 = -1$.

19. Find the CNF of the Boolean expression $E(x, y) = (x \vee \bar{y}) \vee (x \wedge \bar{y})$.

20. Find the remainder when 41^{65} when divided by 7.

21. If $[L, -, \vee, \wedge]$ is a Boolean algebra, then for any a and b in L prove that $\overline{a \wedge b} = \bar{a} \vee \bar{b}$. State its dual.

22. Define Group, Ring and Integral domain.

23. If A={1,2,3,4} and R= $\{(1,2)(2,4)(1,3)(3,2)\}$, then find the transitive closure of R by Warshall's algorithm.

24. If $R = \{(1,1)(2,.2)(2,3)(3,2)(4,2)(4,4)\}$ is a relation on the set S={1,2,3,4} then
   1] Draw its directed graph.
   2] Write its matrix representation.
   3] Find $R^2 = R \circ R$.

25. Find G.C.D of 1357 and 1166 and express it in the form m(1357) + n(1166).

26. State Format's theorem and using Format'\s theorem find $3^{31} = 1(mod 7)$.

27. If A={1,2,3,4} and R= $\{(1,2)(2,4)(3,3)(3,2)\}$, then find the transitive closure of R by Warshall's algorithm.

28. Find Z_4 for addition of residue classes of Residue classes of modulo n.

29. Find Z_5 for multiplication of residue classes of Residue classes of modulo n.

30. Define the term,
   1] Lattice
   2] Distributive Lattice
   3] Complimented Lattice

31. Define Congruence relation modulo n and Let n be positive integers at a, b, c, d are integers prove that....

a] If $a \equiv b(modn)$ then $a + b \equiv b + a(modn)$

b] If $c = d(modn)$ then $ac \equiv bd(modn)$

32. Define Distributive lattice and Complimented lattice, Show that if meet operation distributes over join operation in a lattice then the join operation also distributes over meet operation and vise versa.

33. Define Partial order relation and Let $A = \{a, b, c, d, e\}$ and $A_1 = \{a, c\},\ A_2 = \{b, d, e\}$ be a partition of set A. Find the equivalence set A and show that equivalence classes.

34. Define Partial order relation and Let $A = \{a, b, c, d, e\}$ and $A_1 = \{a, c\},\ A_2 = \{b, d, c\}$ be a partition of set A. Find the equivalence set A and show that equivalence classes.

35. Define Equivalence relation, Let R be an relation on Z, defined by $xRy \Rightarrow 11|5x + 6y$ then show that R is an equivalence Relation

36. If a and b are any two integers and $n \in N$. Then prove that $a \equiv b(modn)$ if and only if a and b leave the same remainder when divided by n

37. Define Lattice, If L is any lattice then for a, b, c ∈ L prove that

a] $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

b] $a \vee (a \wedge b) = a$

38.Define Partial Order relation R on a set X. If R be an partial order relation on X, then show that $x, y \in X, x \in [y]$ if and only if $[x] = [y]$

39. If $d = gcd(a, b)$ then

a] d= ma + nb for some integers m,n

b] If c|b, c|b then c|d

39. State Euclidean algorithm and find the gcd of 243 and 198 for Euclidean algorithm.

40. Show that the Congruence relation is an equivalence relation.

41. Let $a \equiv b(modn)$

Then 1] $ax \equiv bx(modn)$

2] $a + b \equiv b + a(modn)$

42. Define Addition of Residue Classes of modulo n and Find Z_5 for addition residue class of modulo n.

43. Define Multiplication of Residue Classes of modulo n and Find Z_5 Multiplication for residue class of modulo n.

44. Define terms

1] Lattice
2] Distributive Lattice
3] Complimented Lattice
4] Partial order relation
5] Equivalence relation