B.Sc. (Part-II) (Semester-IV) (CBCS) Examination, June-2022

MATHEMATICS

DSC-5D PAPER-VII (Real Analysis-II)

Subject Code: 78907

Question Bank

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Multiple choice Questions

1. If $C = \{C_n\} = \{\sqrt{n}\}$ and $N = \{n_i\} = \{i^4\}, i \in N,$ then $C \circ N = ...$

   a) $\{i\}$ b) $\{i^2\}$ c) $\{i^3\}$ d) $\{i^4\}$

2. If $0 < x < 1,$ then the sequence $\{x^n\}_{n=1}^{\infty}.....$

   a) converges to 0
   b) converges to 1
   c) diverges to ∞
   d) diverges to $-\infty$

3. The sequence $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ is …

   a) non-decreasing
   b) monotonic
   c) bounded
   d) all the above

4. The sequence $\{(-1)^n\}_{n=1}^{\infty}$ is …

   a) not bounded above
   c) monotonic
   c) not bounded below
   d) not monotonic

5. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n-1} = ...$

   a) e b) e^2 c) e^3 d) $\sqrt{e}$

6. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n\pm 50} = ...$

   a) e b) e^2 c) e^3 d) $\sqrt{e}$

7. Every convergent sequence is ….

   a) bounded above
   b) bounded below
   c) bounded
   d) none of these

8. A non-increasing sequence which is bounded below is ….

   a) divergent
   b) oscillatory
   c) convergent
   d) none of these

9. A non-decreasing sequence which is not bounded above …

a) converges to 0
b) converges to 1
c) diverges to ∞
d) diverges to $-\infty$

10. If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative real numbers and $\lim_{n\to\infty} S_n = L$, then …

a) $L = 0$
b) $L = 1$
c) $L \leq 0$
d) $L \geq 0$

11. If $\{S_n\} = \{(-1)^n\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} Sup\, S_n = ...$

a) 0
b) 1
c) -1
d) 2

12. If $\{S_n\} = \{(-1)^n\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} Inf\ S_n = ...$

a) 0
b) 1
c) -1
d) 2

13. If $\{S_n\} = \{1,2,3,4,1,2,3,4,....\}$, then $\lim_{n\to\infty} Sup\, S_n = ...$ and $\lim_{n\to\infty} Inf\ S_n = ...$

a) 4, 1
b) 1, 4
c) 4, 4
d) 1, 1

14. If $\{S_n\}_{n=1}^{\infty}$ is not bounded above, then $\lim_{n\to\infty} Sup\, S_n = ...$

a) 1
b) -1
c) ∞
d) $-\infty$

15. The sequence $\{S_n\} = \{1,0,1,0,1,0,....\}$ is (C, 1) summable to ….

a) 1
b) ½
c) 2
d) 0

16. A sequence $\{S_n\} = \{(-1)^n\}$ is (C, 1) summable to ….

a) 0
b) 1
c) 2
d) -2

17. A sequence $\{S_n\}_{n=1}^{\infty} = \{1\}$ is (C, 1) summable to ….

a) 0
b) 1
c) -1
d) 2

18. The sequence $\{a_n\}_{n=1}^{\infty}$ where, $a_n = \dfrac{n^2+1}{2n^2+5}$, converges to ….

a) 1/5
b) ½
c) 3/2
d) 1

19. A lower bound of the sequence $\{2+n^2\}_{n=1}^{\infty}$ is …

a) 1
b) 4
c) e
d) do not exists

20. The sequence $\{1,1,1,1,....\}$ is ….

a) (C, 1) summable
b) divergent
c) not (C, 1) summable
d) none of these

21. The sequence $\{(-1)^n n\}_{n=1}^{\infty}$ is ….

a) bounded above
b) bounded below
c) bounded above as well as bounded below
d) neither bounded above nor bounded below

22. The supremum of the set $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$ is ….

a) 0
b) 1
c) ½
d) ¼

23. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then ….

a) $\lim_{n\to\infty} \sup[s_n + t_n] \neq \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$

b) $\lim_{n\to\infty} \sup[s_n + t_n] \geq \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$

c) $\lim_{n\to\infty} \sup[s_n + t_n] \leq \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$

d) $\lim_{n\to\infty} \sup[s_n + t_n] \geq \lim_{n\to\infty} \inf[s_n + t_n]$

24. The sequence $\{x_n\}_{n=1}^{\infty}$ converges to 0 if ….

a) $1 \leq x < \infty$

b) $1 < x < \infty$

c) $0 < x \leq 1$

d) $0 < x < 1$

25. A necessary condition for the convergence of the infinite series $\sum u_n$ is ….

a) $\lim_{n\to\infty} u_n \neq 0$

b) $\lim_{n\to\infty} u_n = 0$

c) $\lim_{n\to 0} u_n = 0$

d) $\lim_{n\to 0} u_n \neq 0$

26. The series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + ...$ is ….

a) convergent

b) divergent

c) neither convergent nor divergent

d) none of these

27. The positive p-series $\sum \frac{1}{n^p}$ is divergent for …..

a) $p < 1$

b) $p \geq 1$

c) $p \leq 1$

d) $p > 1$

28. A positive term series converges iff the sequence of partial sums is ….

a) bounded below

b) bounded above

c) both a) and b)

d) none of these

29. Consider the statements (i) $\sum \frac{1}{n}$ is divergent (ii) $\sum \frac{1}{n^2}$ is convergent then ….

a) both (i) and (ii) are true

b) both (i) and (ii) are false

c) only (i) is true

d) only (ii) is true

30. The series $\sum \left(\frac{3}{2}\right)^n$ is ….

a) convergent

b) divergent

c) neither convergent nor divergent

d) none of these

31. If $\sum u_n$ is series of positive terms with $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = L$, then the series diverges if ….

a) L<1

b) L>1

c) L=0

d) none of these

32. The series $\sum \sin \frac{1}{n^2}$ is ….

a) convergent
b) divergent
c) cannot be decided
d neither a) nor b)

33. The series $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ converges to ….

a) e b) 0 c) $\frac{1}{e}$ d) 1

34. The series $\sum\frac{x^n}{n(n+1)}, x>0$ converges for ….

a) $x<1$ b) $x>1$ c) $x\geq 1$ d) $x\leq 1$

35. The series $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...$ is ….

a) divergent
b) convergent
c) neither a) nor b)
d) cannot be decided

36. The series $\sum\frac{n!}{n^n}$ converges to …

a) e b) 1 c) $\frac{1}{e}$ d) 0

37. The series $\sum\frac{1}{\sqrt{n}}$ is ….

a) converges to 0
b) converges to 1
c) divergent
d) none of these

38. For the convergence of series $\sum u_n$, the condition $\lim\limits_{n\to\infty} u_n = 0$ is …

a) only necessary
b) sufficient
c) necessary and sufficient
d) none of these

39. The series $\sum\cos\left(\frac{\pi}{n}\right)$ is ….

a) convergent
b) divergent
c) neither a) nor b)
d) cannot be decided

40. The series $\sum\frac{1}{\sqrt{n+1}-\sqrt{n}}$ is ….

a) converges to 0
b) converges to 1
c) divergent
d) none of these

41. The series $\sum\frac{1}{2n^2+3n+5}$ is ….

a) convergent
b) divergent
c) neither a) nor b)
d) cannot be decided

42. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if ….

a) $p < 1$ b) $p \geq 1$ c) $p \leq 1$ d) $p > 1$

43. The geometric series $\sum_{n=1}^{\infty} x^{n-1}$ is convergent if ….

a) $x > 1$ b) $0 \leq x < 1$ c) $x \neq 0$ d) $x = 1$

44. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is ….

a) convergent b) divergent c) oscillatory d) none of these

45. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is ….

a) convergent b) divergent c) oscillatory d) none of these

46. If $\sum_{n=1}^{\infty} a_n$ is a series of positive terms and $\lim_{n\to\infty} \sqrt[n]{a_n} = l$, then the series is convergent if ….

a) $l > 0$ b) $l = 1$ c) $l > 1$ d) $l < 1$

47. A series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if …. is convergent.

a) $\left\{\sum_{k=1}^{\infty} a_k\right\}_{n=1}^{\infty}$ b) $\left\{\sum_{k=1}^{\infty} a_k{}^2\right\}_{n=1}^{\infty}$ c) $\{a_k\}_{k=1}^{\infty}$ d) $\{|a_n|\}_{n=1}^{\infty}$

48. If $\sum u_n$ and $\sum v_n$ are two positive term series that $\lim_{n\to\infty} \frac{u_n}{v_n} = l$ is non-zero finite number, then …..

a) if $\sum u_n$ converges then $\sum v_n$ converges
b) if $\sum v_n$ converges then $\sum u_n$ converges
c) if $\sum u_n$ diverges then $\sum v_n$ converges
d) both series converges or diverges together

49. The sum of the series $\sum_{n=0}^{\infty} \frac{1}{n!}$ is ….

a) e b) 0 c) 1 d) -1

50. The series 0.6+0.06+0.006+…. is ….

a) convergent to 0.6
b) convergent to 2/3
c) convergent to 1/3
d) divergent

Long Answer Questions (10 Mark Questions)

1. Define convergent sequence and prove that a non-decreasing sequence which is bounded above is convergent.
2. Define convergent sequence and prove that a non-increasing sequence which is bounded below is convergent.
3. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.
4. If $\{s_n\}_{n=1}^{\infty}$ is sequence of real numbers which converges to L then prove that $\{s_n^2\}_{n=1}^{\infty}$ converges to L^2.
5. If $0 < x < 1$, then prove that the sequence $\{x^n\}_{n=1}^{\infty}$ converges to zero.
6. If $\{s_n\}_{n=1}^{\infty}$ is sequence of real numbers then prove that $\lim_{n\to\infty} \sup s_n = \lim_{n\to\infty} s_n$.
7. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then prove that
   i) $\lim_{n\to\infty} \sup (s_n + t_n) \le \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$ and
   ii) $\lim_{n\to\infty} \inf (s_n + t_n) \ge \lim_{n\to\infty} \inf s_n + \lim_{n\to\infty} \inf t_n$
8. If $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence of real numbers then prove that $\{s_n\}_{n=1}^{\infty}$ is convergent.
9. Prove that a positive term geometric series $\sum_{n=0}^{\infty} r^n$ converges for $r < 1$ and diverges to $+\infty$ for $r \ge 1$.
10. Prove that a positive term series $\sum \frac{1}{n^p}$ is convergent for $p > 1$ and diverges for $p \le 1$.
11. If the alternating series $u_1 - u_2 + u_3 - u_4 + \ldots (u_n > 0 \; \forall n)$ is such that
    i) $u_{n+1} \le u_n \; \forall n$ and ii) $\lim_{n\to\infty} u_n = 0$. then prove that the series convergent.
12. State D'Alembert's ratio test. Hence test the convergence of $x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots$
13. State Raabe's test. Hence discuss the convergence of
    $$\frac{(1!)^2}{2!} x + \frac{(2!)^2}{4!} x^2 + \frac{(3!)^2}{6!} x^3 + \ldots \quad (x > 0)$$
14. State Cauchy's root test. Hence test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+2}\right)^{n^2}$.
15. Define absolute and conditional convergence. Hence show that the series
    $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ converges absolutely for all values of x.

Short Answer Questions (5 mark questions)

1. Find the limit of the sequence 0.6, 0.66, 0.666, ….
2. Find the limit of the sequence $\{s_n\}_{n=1}^{\infty}$ where $s_n = \sqrt[3]{5}$, $s_{n+1} = \sqrt[3]{5 \cdot s_n}$.
3. Find the limit superior and inferior of the sequence 10^{100}, 1, -1, 1, -1, …
4. Find the limit superior and inferior of the sequence 1, 2, 3, 4, 1, 2, 3, 4,….
5. Find the limit superior and inferior of the sequence $\left\{\sin\left(\frac{n\pi}{2}\right)\right\}_{n=1}^{\infty}$
6. Prove that sequence $\{s_n\} = (-1)^n$, $(n \in I)$ is (C, 1) summable.
7. Prove that the sequence 1, 0, 1, 0, 1, 0, …. is (C, 1) summable.
8. Discuss the (C, 1) summability of 1, -1, 2, -2, 3, -3, …..
9. Prove that the sequence $s_n = 1, 1, 1, 1, ...$ is (C, 1) summable.
10. Discuss the (C, 1) summability of 1, 2, 3, ….
11. Show that the sequence $\{s_n\}$, where $s_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}$, $\forall n \in N$ is convergent.
12. Show that the sequence $\{s_n\}$, where $s_n = \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}$, $\forall n \in N$ is convergent.
13. Show that the sequence $\{s_n\}$ defined by the recursion formula $s_{n+1} = \sqrt{3s_n}$, $s_1 = 1$ converges to 3.
14. Find the limit superior and inferior of the sequence $s_n = 1 + (-1)^n$, $n \in N$
15. Find the limit superior and inferior of the sequence $s_n = \sin\left(\frac{n\pi}{3}\right), n \in N$
16. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(3n+1)(3n+4)}$.
17. Find sum of the series $\sum_{n=1}^{\infty} \frac{1}{16n^2 + 8n - 3}$.
18. Find the sum of the series $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + ...$
19. Test the convergence of the series and find the sum if the series convergent

    $0.3 + 0.03 + 0.003 + ...$
20. Discuss the convergence of the series for $x = \frac{3}{4}$:

    $1 + 2x + x^2 + 2x^3 + x^4 + 2x^5 + ...$

21. Discuss the convergence of the series $\frac{2}{3}-\frac{4}{9}+\frac{8}{27}-\frac{16}{81}+...$

22. Test the convergence of the series $\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+...$

23. Show that the series $\frac{1\cdot 2}{3^2\cdot 4^2}+\frac{3\cdot 4}{5^2\cdot 6^2}+\frac{5\cdot 6}{7^2\cdot 8^2}+...$ is convergent.

24. Investigate the behavior of the series $\sum \sin\frac{1}{n}$

25. Discuss the convergence of $\sum \frac{8+5^n}{3+4^n}$

26. Test the convergence of the series $\sum_{n=1}^{\infty}\frac{3^n}{n^3}$

27. Test the convergence of the series $\sum \frac{3^n}{n^3+3}$

28. Test the convergence of $\frac{3}{7}+\frac{3\cdot 6}{7\cdot 10}+\frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}+...$

29. Test the convergence of $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...$

30. Show that $\sum \frac{\sin n\alpha}{n^2}$ is absolutely convergent.

B.Sc. (Part-II) (Semester-IV) (CBCS) Examination, June-2022

MATHEMATICS

DSC-6D PAPER-VIII (Algebra-II)

Subject Code: 78907

Question Bank

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Multiple choice Questions

1. If H is a subgroup of a finite group G and o(H)=3, o(G)=36 then [G:H]=……
   a) 3 b) 12 c) 4 d) 9
2. The theorem that for any integer a and prime $p, a^p \equiv a(\text{mod } p)$ is called ….
   a) Fermat's theorem c) Sylow's first theorem
   b) Euler's theorem d) Lagrange's theorem
3. If G is a finite group and H is a subgroup of G then $i_G(H) = ...$
   a) $\frac{o(H)}{o(G)}$ b) $o(G)$ c) $o(H)$ d) $\frac{o(G)}{o(H)}$
4. If G is a finite group and $a \in G$, then ….
   a) $o(G) | o(a)$ b) $o(a) > o(G)$ c) $o(a) | o(G)$ d) none of these
5. If H is a subgroup of a finite group G then …..
   a) $[G:H] | o(H)$ b) $[G:H] | o(G)$ c) $o(G) | [G:H]$ d) $o(H) | [G:H]$
6. The theorem that "If a is any integer relatively to positive integer n" then $a^{\phi(n)} \equiv 1(\text{mod } n)$, where ϕ is the Euler ϕ-function is called ….
   a) Lagrange's theorem c) Euler's theorem
   b) Fermat's theorem d) Sylow's theorem
7. If His a subgroup of a finite group G, then index of H in G is …
   a) $o(H)$ b) $\frac{o(G)}{o(H)}$ c) $\frac{o(H)}{o(G)}$ d) $o(G)$
8. A subgroup H of a group G is normal if and only if ….. for all $g \in G$.
   a) $Hg^{-1} = H$ b) $gH = H$ c) $gHg^{-1} = H$ d) $Hg^{-1} = gH$
9. A subgroup H of a group G is normal if and only if …..
   a) Every right coset of H in G is greater than a left coset of H in G.
   b) Every right coset of H in G is a left coset of H in G.
   c) Every left coset of H in G is greater than a right coset of H in G.
   d) None of these.

10. Every subgroup of abelian group is ….

a) a normal subgroup
b) abelian but not normal
c) neither abelian nor normal
d) none of these

11. A subgroup H of a group G is normal in G if and only if for any $a,b \in G$, ….

a) $(Ha / Hb) = Hab$
b) $Ha + Hb = Hab$
c) $(Ha)(Hb) = H_{a+b}$
d) $(Ha)(Hb) = Hab$

12. Let H be a subgroup and K be normal subgroup of the group G, then … is normal in H.

a) $H \cup K$
b) $H \cap K$
c) H+K
d) none of these

13. If G is a group and $a \in G$, then the subset $\{x \in G \mid xa = ax\}$ is called ….

a) normalizer of a in G
b) center of G
c) right coset of a in G
d) none of these

14. If G is a group then $\{x \in G \mid xg = gx,\ for\ all\ g \in G\}$ is called ….

a) normalizer of a in G
b) center of G
c) right coset of a in G
d) none of these

15. If G is a finite group and N is a normal subgroup of G then $o\left(\frac{G}{N}\right)$ is ….

a) $\frac{o(G)}{o(N)}$
b) $\frac{o(N)}{o(G)}$
c) $2 \cdot o(G)$
d) $2 \cdot o(H)$

16. The factor group of an abelian group is ….

a) cyclic
b) neither abelian nor cyclic
c) abelian
d) none of these

17. The factor group of a cyclic group ….

a) cyclic
b) neither abelian nor cyclic
c) abelian
d) none of these

18. If G is finite group and H is subgroup of G then index of H in G is ….

a) $\frac{o(H)}{o(G)}$
b) $o(G)$
c) $o(H)$
d) $\frac{o(G)}{o(H)}$

19. If H is a subgroup of a finite group G and order of H and G are respectively m and n then …

a) $m \mid n$
b) $n \mid m$
c) $m = kn, k \in Z$
d) m does not divides n

20. Let H be a subgroup of finite group G, then for any $x \in G$ ….

a) $o(x^{-1}Hx) = o(H)$
b) $o(Hx) = o(xH)$
c) $(x^{-1}Hx) = H$
d) $Hx = xH$

21. If o(G)=30 then group G may have subgroup of order ….

a) 4
b) 5
c) 7
d) 8

22. Any two right cosets of a subgroup H of a group G are ….

a) either disjoint or identical
c) always identical

b) always disjoint d) left cosets

23. A group having no non-trivial normal subgroups then group G is known as ….
    a) alternating group c) conjugate group
    b) simple group d) symmetric group
24. For Euler's function ϕ if n=8 then $\phi(n) = ...$
    a) 4 b) 6 c) 7 d) 8
25. If p is a prime the $\phi(p) = ...$
    a) p b) p-1 c) p+1 d) p/(p-1)
26. Order of …. Group is equal to order of its generator.
    a) simple b) abelian c) cyclic d) commutative
27. The remainder of 3^{12} when divided by 7 is ….
    a) 3 b) 1 c) 0 d) 7
28. The remainder of 2^{14} when divided by 7 is ….
    a) 4 b) 1 c) 0 d) 7
29. The remainder of 7^{17} when divided by 17 is ….
    a) 17 b) 1 c) 0 d) 7
30. Let $f: G \to G'$ be a homomorphism of a group G into G' with kernel K. Then f is one-one iff ….
    a) $K = \phi$ b) $K = \{e\}$ c) $K = G$ d) $K = G'$
31. A homomorphism f from a group G onto the group G' is an isomorphism iff kerf=….
    a) ϕ b) $\{e\}$ c) $K = G$ d) $K = G'$
32. Every homomorphic image of a group G is isomorphic to ….
    a) a permutation group c) a quotient group
    b) a cyclic group d) simple group
33. Order of the permutation (1 2 4 5) (3 6) is ….
    a) 2 b) 3 c) 4 d) 6
34. If $f: G \to G'$ be an isomorphism and a be any element of G then ….
    a) o(G)=o(a) b) o[f(a)]=o(a) c) o(G')=o(a) d) o(a)=o(G/G')
35. Any infinite cyclic group is isomorphic to …..
    a) the group R of real numbers c) the group Z of integers
    b) the group Q of rational numbers d) the set R^+ of positive rational numbers.
36. Identity permutation is always …. Permutation.
    a) odd b) positive c) negative d) even
37. The inverse of an even permutation is …..
    a) even permutation c) odd permutation
    b) transposition d) cycle
38. The inverse of an odd permutation is …..
    a) even permutation c) odd permutation
    b) transposition d) cycle

39. Every finite group is isomorphic to ….

a) normal group
b) quotient group
c) cyclic group
d) permutation group

40. The order of symmetric group S_3 is ….

a) 3
b) 3^2
c) 6
d) 1

41. The symmetric group S_3 is ….

a) finite and abelian
b) infinite and abelian
c) finite and non-abelian
d) infinite and non-abelian

42. Every finite group is isomorphic to permutation group is ….

a) true
b) false
c) either true or false
d) neither true nor false

43. "Every finite group G is isomorphic to a permutation group" is the statement of ….

a) Cauchy's theorem
b) Euler's theorem
c) Leibnitz's theorem
d) Cayley's theorem

44. A one-one onto mapping $f : S \to S$ is called ….

a) isomorphism
b) function
c) monomorphism
d) permutation

45. One-one homomorphism is called ….

a) isomorphism
b) monomorphism
c) epimorphism
d) endomorphism

46. Let R be a ring. An element $0 \neq a \in R$ is called a zero divisor if there exists an element $0 \neq b \in R$ such that …

a) $ba \neq 0$
b) $ba = 0$
c) $ab = a$
d) $ba = b$

47. A ring R is called a …. , if $x^2 = x$ for all $x \in R$.

a) Ring with unity
b) division ring
c) Boolean ring
d) field

48. Any ring can be imbedded into ….

a) Ring with unity
b) ring without unity
c) ring without zero divisor
d) ring with zero

49. An element x in a ring R is called nilpotent if …

a) $x^n = x$
b) $x^n = 0$
c) $x^n = e$
d) $x^n = 1$

50. An element e in a ring R such that $e^2 = e$ is known as ….

a) Identity
b) nilpotent
c) idempotent
d) unity

Long Answer Questions (10 Mark Questions)

1) If H is a subgroup of a finite group G then prove that o(H) divides o(G). Is converse true ?

2) Prove that "A subgroup H of a group G is normal in G if and only if the product of any two right (or left) cosets H in G is again a right (or left) coset of H in G."

3) Define centre of group G and prove that centre Z(G) of a group G is a normal subgroup of G.

4) Define quotient group $\frac{G}{H}$ containing all cosets of the form H_a and show that $\frac{G}{H}$ is a group under the binary operation $H_a . H_{b} = H_{ab}$ where H is a normal subgroup of group G.

5)If H and K be two subgroups of a group G then define nonempty subset HK and prove that HK is a subgroup of G if and only if HK = KH.

6) If f : G→ $G^{/}$ is an onto homomorphism with K = Ker f, then show that $\frac{G}{K} \cong G^{/}$.

7)Define Kernel of homomorphism and prove that if f : G → $G^{/}$ is a homomorphism, then Ker f is a normal subgroup of G.

8)State fundamental theorem of group homomorphism and using this prove that any finite cyclic group of order n is isomorphic to the quotient group $\frac{Z}{N}$, where (Z, +) is the group of integers and N =(n).

9) Show that the set S_n of all permutations of degree n defined on a non-empty finite set S of n elements is a finite non-abelian group of order n! under permutation multiplication.

10)Prove that, Every finite group G is isomorphic to a permutation group.

11) Prove that any finite cyclic group of order n is isomorphic to additive group of integers modulo n.

12) if f : G → $G^{/}$ be an onto homomorphism from group G to $G^{/}$. Let H be a subgroup of G and $H^{/}$ be a subgroup of $G^{/}$. Then prove that (i) f(H) is a subgroup of $G^{/}$ (ii) $f^{-1}(H^{/})$ is a subgroup of G containing K = Ker f.

13)Define ring, commutative ring and show that the set Z_7 ={0, 1, 2, 3, 4, 5, 6} forms a commutative ring with unity under addition and multiplication modulo 7.

14) Define subring of a ring R and prove that, a non-empty subset S of a ring R is a subring of R if and only if a, b ∈ S ⇒ ab, a-b ∈ S.

15) Define right ideal, left ideal, ideal of a ring R and show that intersection of two ideals of a ring R is again an ideal of R.

Questions for 5 Marks

1) If G is a finite group and a ∈ G, then show that o(a) divides o(G).

2) If G is a finite group of order n then prove that for all a ∈ G, $a^n = e$, where e is the identity element of G.

3) State and prove Fermat's theorem.

4) Define Euler's ϕ function and find ϕ(8).

5)Using Fermat's theorem compute the remainder of 2^{35} when divided by 7.

6) Using Fermat's theorem compute the remainder of 3^{31} when divided by 7.

7) Prove that a group H of a group G is normal if and only if every right coset of H in G is a left coset of H in G.

8) Prove that every sub group of an abelian group is a normal subgroup.

9) If G is a group and N, M are two normal subgroups of G then show that N ∩ M is also normal subgroup of G.

10)Show that if N and M are two normal subgroups of a group G then NM is also normal subgroup of G.

11) Prove that factor group of a cyclic group is cyclic.

12) Prove that factor group of an abelian group is abelian.

13) If N is normal subgroup of a group G and M is a sub group of G then prove that NM is sub group of G.

14) Let G ={0, 1, 2, 3, 4, 5} and H = {0, 3}. Then find quotient group $\frac{G}{H}$.

15) Let f be a mapping from (Z, +), the group of integers to the group G = {1, -1} under multiplication defined as f(x) = 1, if x is even and f(x) = -1, if x is odd. Then show that f is a homomorphism.

16)Find fg and gf if f = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 4 & 3 & 8 & 6 & 7 & 1 \end{pmatrix}$ and g = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 6 & 5 & 1 & 8 & 3 & 2 & 4 \end{pmatrix}$

17)Find inverse of permutation

f = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ and show that f f^{-1} =1

18) Let f = $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and g = $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ be two permutations of degree 3. Show that fg ≠ gf

19)Let S = {1, 2, 3}. Then write the set P_3 of all permutations of degree 3.

20)If f : G→ $G^/$ is a homomorphism then show that f(e) = $e^/$, where e and $e^/$ are the identity elements of groups G and $G^/$ respectively.

21) If f : G→ $G^/$ is a homomorphism then show that range of f is a subgroup of $G^/$.

22) If a, b are any elements of a ring R, Prove that a.(-b) = (-b).a = -ab

23) If a, b are any elements of a ring R, Prove that (-a).(-b) = a.b

24) Show that multiplicative identity in a ring R if exists then it is unique.

25) Define zero-divisor in a ring R and find zero divisor in a ring (Z_6 , $\oplus_6$, $\odot_6$)

26) Show that the set of numbers of the form a+b$\sqrt{2}$, with a and b as rational numbers with usual addition and multiplication is a field.

27) Show that the set N of all 2 × 2 matrices of the form $\begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$ for a, b integers forms a left ideal in the ring R of all 2 × 2 matrices with elements as integers.

28) Show that the set N of all 2 × 2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for a, b integers forms a right ideal in the ring R of all 2 × 2 matrices with elements as integers.

29) Let R be the ring of integers. Let m be any fixed integer and let S be any subset of R such that S = {…, -3m, -2m, -m, 0, m, 2m, 3m, …}. Then show that S is a subring of R.

30) Show that the set of matrices $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ is a subring of the ring of 2 × 2 matrices with integral elements.