Shivaji University, Kolhapur

Question Bank For

B.Sc. (Part –III) (Semester –VI) Examination, March/April-2022

MATHEMATICS

Complex Analysis (Paper-XV) Sub. Code: 81664

Multiple Choice Questions

1 Functions satisfying Laplace equation are known as.......

a) Homomorphic

b) conjugate

c) harmonic

d) regular

2 If f(z) is analytic and equals $u(x,y)+iv(x,y)$ then $f'(z)=$

a) $\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}$

b) $\frac{\partial v}{\partial y}-i\frac{\partial v}{\partial x}$

c) $\frac{\partial u}{\partial x}-i\frac{\partial v}{\partial x}$

d) $\frac{\partial v}{\partial y}+i\frac{\partial v}{\partial x}$

3 An analytic function with constant modulus is

a) variable

b) zero

c) does not exist

d) constant

4 Let $f(z)=|z|^2$ then which of the following statement is correct

a) is continuous everywhere

b) f is differentiable at $z\neq 0$

c) f is not analytic at $z = 0$

d) is not continuous everywhere

5 The value of m so that $2x - x^2 + my^2$ may be harmonic is

a) 0

b) 1

c) 2

d) 3

6 The value of integral $\int_0^{1+i} z dz$ is

a) 0

b) i

c) $1 + 2\text{i}$

d) $1 - 2\text{i}$

7 Harmonic conjugate of $\text{u}(\text{x}, \text{y}) = e^x \cos y$ is

a) $e^y \cos x$

b) $e^x \cos y$

c) $e^x \sin y + c$

d) $\frac{e^y}{\sin x}$

8 The value of contour integral $\int_c \frac{z^3}{(z-2)^2} dz$ over $|z| = 1$ is

a) $24\pi i$

b) $12\pi i$

c) $6\pi i$

d) 0

9 Polar form of Cauchy Riemann equations are.

a) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \ \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$

b) $\frac{\partial v}{\partial \theta} = \frac{1}{r}\frac{\partial u}{\partial r}, \ \frac{\partial u}{\partial r} = \frac{-1}{r}\frac{\partial v}{\partial \theta}$

c) $\frac{\partial u}{\partial r} = r\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -r\frac{\partial u}{\partial \theta}$

d) $\frac{\partial u}{\partial r} = r\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$

10 If C is the circle $|z-a|=r$ then $\int_C \frac{dz}{(z-a)^n}=2\pi i$ when,..

a) $n=1$

b) $n\neq 1$

c) $n=0$

d) $n=\infty$

11 The order of the pole of the function $f(z)=\frac{z^4+2z+1}{z^2+5z+2}$ at $z=\infty$ is ....

a) 2

b) 1

c) 0

d) 4

12 The complex function $f(z)=e^z$ at the point $z=\infty$ is .....

a) a pole of order 1

b) a pole of order 2

c) an isolated essential singularity

d) a non-isolated essential singularity

13 The value of $\int_C \frac{dz}{z+2}$ where c is $|z|=1$ is

a) $2\pi i$

b) $-2\pi i$

c) $4\pi i$

d) 0

14 Residue of $f(z)=\int_C \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z=3$ is

a) $\frac{101}{16}$

b) -8

c) $\frac{27}{16}$

d) 0

15 If $\omega=u+iv$ be analytic function of $z=x+iy$ then the families of curves $u(x,y)=c_1 \& v(x,y)=c_2$ form.

a) Orthogonal system

b)Conjugate system

c) Harmonic system

d) Analytic system

16 The value of the integral $\int_c \frac{dz}{z-a}$ where c is given by the equation $|z-a| = \mathrm{R}$ is

a) $2\pi i$

b) $\frac{\pi i}{2}$

c) $-\pi i$

d) $-\frac{\pi i}{2}$

17 The Singular Point of the function $f(z) = \frac{z+4}{z^2-9z+20}$ are.

a) $z = 4,5$

b) $z = -4,5$

c) $z = 4,-5$

d) $z = -4,-5$

18 A point at which a function $f(z)$ ceases to be analytic is called.

a) zero

b) singularity

c) pole

d) limit point

19 $f(z) = \bar{z}$ is

a) Continuous for every z, not differentiable for any z.

b) Continuous for some values of z, differentiable for every z.

c) discontinuous for every z, differentiable for every z.

d) neither continuous nor differentiable.

20 If a function $f(z)$ is analytic in the entire complex plane then $f(z)$ is called...........

a) Analytic continuation

b) Meromorphic function

c) Integral function

d) Harmonic function

21 An analytic function with constant modulus is

a) variable

b) constant

c) is zero

d) does not exist

22 A continuous arc without multiple points is called a

a) Jordan arc

b) continuous arc

c) contour

d) rectifiable arc

23 If an entire function has a pole of order five at infinity then the polynomial of degree ....

a) 4

b) 5

c) 6

d) 0

24 To evaluate the integrals of the type $\int_0^{2\pi} \emptyset(\cos\theta, \sin\theta)$, the contour used is.

a) Any circle

b) semi circle

c) unit circle

d) rectangle

25 If principal part of Laurent's series consists of no terms then singularity is called ....

a) removal Singularity

b) pole

b) essential Singularity

d) irremovable Singularity

26 The value of $\int_c \frac{dz}{z}$, where c is the circle with centre at the origin and radius r is

a) $\log r$

b) πi

c) $2\pi i$

d) $\frac{\pi i}{2}$

27 Every analytic function in a simply connected domain

a) Possesses a definite integral

b) Possesses a indefinite integral

c) Does not Possesses an indefinite integral

d) None of these

28 If $f(z) = u + iv$ to be analytic then Cauchy-Riemann equations are..............

a) $u_x = v_y, u_y = -v_x$

b) $u_x = -v_y, u_y = v_x$

c) $u_x = v_y, u_y = v_x$

d) $u_x = -v_y, u_y = -v_x$

29 If c is the circle $|z - a| = r$, then $\int_c \frac{1}{(z-a)^n} dz = 2\pi i$, when

a) $n \neq 1$

b) $n = 0$

c) $n = 1$

d) none of these

30 If $f(z) = u + iv$ is analytic function then

a) both u & v are harmonic functions

b) both u & v need not to be harmonic function

c) u is harmonic function but v is not harmonic function

d) v is harmonic function but u is not harmonic function

31 If in the principal part of Laurent's series consist of an infinite number of terms then singularity is called......

a) removable singularity

b) essential singularity

c) pole

d)non-essential singularity

32 If f(z) has a pole at $z = a$, then as $z \to a$,

a) $|f(z)| \to 0$

b) $|f(z)| \to a$

c) $|f(z)| \to \infty$

d) $|f(z)| \to -\infty$

33 A function whose only singularities in the entire complex plane are poles, is called ....

a) analytic function

b) homomorphic function

c) memomorphic function

d) regular function

34 Function e^z has at $z = \infty$ is

a) isolated singularity

b) a pole

c) an infinite point

d) an isolated essential singularity

35 What kind of a) simple pole

function $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$ is

b) double pole

c) singularity

d) none of these

36 Residue of $f(z)$ at a simple pole $z = a$ is ....

a) $\lim_{z \to a} z f(z)$

c) $\lim_{z \to a} (z - a) f(z)$

b) $\lim_{z \to a} \frac{f(z)}{z-a}$

d) $\lim_{z \to a} \frac{z-a}{f'(z)}$

37 Residue of $\frac{z^2-2z}{(z+1)^2(z^2+4)}$ at double pole as $z = -1$ is

a) $4/5$

b) $-4/5$

c) $-14/25$

d) $14/25$

38 To integrate $\int_{-\infty}^{\infty} f(x)dx$ we should use

a) circular contour

b) indented semi-circular

c) rectangular contour

d) unit circle

39 Residue of f(z) at $z = \infty$ is

a) $\frac{1}{2\pi i}\int_c f(z)dz$

b) $-\frac{1}{2\pi i}\int_C f(z)dz$

c) $2\pi i\int_c f(z)dz$

d) $-2\pi i\int_c f(z)dz$

40 The value of $\int_C \frac{e^z}{z-2}$ at $|z| = 1$ is

a) $2\pi i$

b) 0

c) $2\pi i z_0$

d) $4\pi i$

41 The value of the integral $\int_c \frac{dz}{z-2}$ where c is $|z| = 3$

a) $2\pi i$

b) $-2\pi i$

c) $-\pi i$

d) $-\frac{\pi i}{2}$

42 Function $f(z) = xy + iy$ is

a)Everywhere continuous and analytic

b) Everywhere continuous but not analytic

c)Discontinuous but analytic everywhere

d) neither continuous nor analytic.

43 A continuous arc without multiple points is called a

a) Jordan arc

b) continuous arc

c) contour

d) rectifiable arc

44 Consider two statements

(i) Functions satisfying Laplace's equation is a harmonic

(ii) An analytic function with constant modulus in a domain is constant

a) Only (i) is true

b) Only (ii) is true

c) Both (i) and (ii) are true

d) Both (i) and (ii) are false

45 To integrate $\int_0^\infty \frac{dx}{1+x^2}$ we will use a contour.

a) real axis & unit circle $|z| = 1$

b) real axis & lower half of circle $|z| = R$

c) real axis & upper half of circle $|z| = R$

d) circle $|z| = R$

46 The limit point of the poles of a function $f(z)$ is

a) a pole

b) an isolated singularity

c) a non isolated singularity

d) a non isolated essential singularity

47 If f(z) is an analytic function of z& if f$'$(z) is continous at each point within & on a closed contour c then ....

a) $\int_c f(z)dz = 2\pi i$

b) $\int_c f(z)dz = 0$

c) $\int_c f(z)dz = \pi i$

d) $\int_c f(z)dz \neq 0$

48 If an entire function has a pole of order 4 at infinity then it is a polynomial of degree

a) 5

b) 4

c) Zero

d) infinity

49 The value of $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ is

a) $2\left(\frac{\partial^2}{\partial z\,\partial \bar{z}}\right)$

b) $4\left(\frac{\partial^2}{\partial z\,\partial \bar{z}}\right)$

c) $-2\left(\frac{\partial^2}{\partial z\,\partial \bar{z}}\right)$

d) $-4\left(\frac{\partial^2}{\partial z\,\partial \bar{z}}\right)$

50 The analytic functions are called

a) isomorphic

b) homomorphic

c) holomorphic

d) conformal

Questions for 8 Marks

1) State and prove necessary conditions for f(z) to be analytic.
2) If $f(z) = u(x, y + iv(x, y)$ is analytic function and $z = re^{i\theta}$ is a polar form of z then show that Cauchy- Riemann equation in polar form are $\frac{\partial u}{\partial r} = \frac{1}{r}\,\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\,\frac{\partial u}{\partial \theta}$
3) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles $z_1, z_2, \ldots, z_n$ within the closed contour and continuous on boundary which is a rectifiable Jordan curve then prove that $\int_C f(z)dz = 2\pi i \sum_{k=1}^{n} Res(z = z_k)$ hence evaluate $\int_C \frac{dz}{z^3(z+3)}$ over the circle $C: |z| = 1.$
4) Explain exact differential equation method for construction of an analytic function. Hence construct analytic function for $u = e^x\, cosy$
5) If $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C, if $|g(z)| < |f(z)|$ on C then show that $f(z)$ and $f(z) + g(z)$ both have same number of zeros inside C.
6) Explain Milne-Thomson Method for construction of an analytic function by considering both cases.
7) If $(z) = \frac{x^3(1+i) - y^3\,(1-i)}{x^2 + y^2}$, $z \neq 0$ and $f(z) = 0,\ z = 0$ then show that $f(z)$ is continuous and satisfy Cauchy Riemann equations at origin and $f'(0)$ does not exist.

8) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles $z_1, z_2, \ldots., z_n$ within the closed contour and continuous on boundary which is a rectifiable Jordan curve then prove that $\int_C f(z)dz = 2\pi i \sum_{k=1}^{n} Res(z = z_k)$ hence evaluate $\int_C \frac{e^{az}}{z+1}\, dz$ over the circle $C: |z| = 2$.

9) State and prove Cauchy theorem for simply connected domain.

10) State and prove sufficient conditions for $f(z)$ to be analytic.

11) If a domain D is bounded by system of closed rectifiable curves $C_1, C_2, \ldots \ldots., C_k$ and $f(z)$ is analytic in domain D and continuous on $C_1, C_2, \ldots \ldots., C_k$ then show that $\int_{C_1} f(z)dz + \int_{C_2} f(z)dz + \ldots \ldots \ldots \ldots \ldots + \int_{C_k} f(z)dz = 0$

12) If $f(z)$ is analytic in domain D then show that $f(z)$ has derivatives of all orders at any point $z = a$ and all of which are analytic in domain D, there values is given by $f^n(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}}\, dz$ where C is any closed curve surrounding the point $z = a$.

13) State and prove Cauchy integral formula for simply connected domain.

14) Explain the method to evaluate the integral of the type $\int_{-\infty}^{\infty} f(z)dz$ hence show that $\int_0^{\infty} \frac{dz}{1+z^2} = \frac{\pi}{2}$.

15) State and prove Cauchy Residue theorem.

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Questions for 4 Marks

1) Show that $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

2) If $f(z) = u + iv$ is an analytic function. If $f'(z) = 0$ then show that $f(z)$ is constant.

3) If $f(z) = u + iv$ is an analytic function then show that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z\ \partial \bar{z}}$.

4) If $u = e^x \, cosy$ is harmonic function then find the harmonic conjugate of u by using exact differential equation method and hence construct the analytic function.

5) Evaluate $\int \bar{z} \, dz$ along the line from $z = 0$ to $z = 2i$ and then from $z = 2i$ to $z = 4 + 2i$.

6) Find the value of integral $\int_0^{1+i} (x - y + ix^2) dz$ along the straight-line from $z = 0$ to $= 1 + i$.

7) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $|z| > 3$.

8) Show that $\int_C \bar{z} \, dz$ along a semi-circular path from $z = -a$ to $z = a$ lies above the x axis is $-\pi i a^2$.

9) If $f(z) = \frac{z-4}{(z-3)\,(z-5)^2}$ then find the residues at corresponding poles.

10) Prove that analytic function with constant modulus is constant.

11) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

12) Show that the real and imaginary part of $f(z) = e^z$ are harmonic.

13) Evaluate $\int_0^{2\pi} \frac{1+2\,cos\theta}{5+4\,cos\theta} \, d\theta$.

14) If $f(z) = u + iv$ is an analytic function then show that $u(x, y) = c_1$ and $v(x, y) = c_2$ represent orthogonal family of curves.

15) Evaluate $\int_0^{1+i} z^2 \, dz$.

16) If $f(z) = u + iv$ is an analytic function then find $f(z)$ in terms of z where $u - v = (x - y)(x^2 + 4xy + y^2)$.

17) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $1 < |z| < 3$.

18) If $f(z) = \frac{z^4}{z^2+a^2}$ then find the residues at corresponding poles.

19) Show that $\int_C \bar{z}\,dz$ along a semi-circular path from $z = -a$ to $z = a$ lies below the x axis is $\pi i a^2$.

20) Find the type of singularities of $f(z) = \frac{cot\pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$.

21) If $u = log(x^2 + y^2)$ then construct the analytic function.

22) Evaluate $\int_C \frac{z^4}{z-3i}\, dz$ by using Cauchy integral formula along the curve $|z - 2| < 5$.

23) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

24) What kind of singularities exist for the function $f(z) = \frac{1-e^z}{1+e^z}$ at $z = \infty$.

25) Find the value of integral $\int_0^{1+i} (x - y + ix^2)dz$ along the straight-line from $z = 0$ to $z = 1 + i$.

26) Find the residue of $f(z) = \frac{z^2}{(z^2+1)^2}$ at $z = \infty$.

27) Expand $f(z) = \frac{1}{z\,(z^2-3z+2)}$ in a Laurent's series for the region $0 < |z| < 1$.

28) If $f(z) = \frac{z-4}{(z-3)\,(z-5)^2}$ then find the residues at corresponding poles.

29) Evaluate $\int_C \frac{e^{az}}{z+1}\, dz$ over the circle $C: |z| = 2$.

30) Find the type of singularities of $f(z) = tan\left(\frac{1}{z}\right)$ at $z = 0$.

31)Write Cauchy- Riemann equations in cartesian form.

32) Define singular point

33)How to calculate the singularity of function f(z) at infinity?

34)Define pole.

35)What is the isolated singularity?

36)Define the Jordan curve.

37)Define complex valued function.

38) Define meromorphic function

39)Define limit of a complex valued function

40)Write Cauchy- Riemann equations in polar form.

41)Define principal argument of complex number

42)Define harmonic function

43)What is the modulus and argument of $z = -i$.

44)Define removable singularity.

45) Define regular point.

46)Define simple curve.

47) Write the solution of exact differential equation $Mdx + Ndy = 0$.

48)Define entire function.

49)Write the equation of circle whose center is at c and radius a in complex plane.

50)Define analytic function.

51)Write statement of Greens theorem.

52)Define essential singularity.

53)Define cross cut.

54)Define smooth curve.

55)Define residue of an isolated singularity.