Shivaji University, Kolhapur
B.Sc. (Part III) (Semester V) Examination June 2022
QUESTION BANK
Paper Code: DSE E12 Subject Code : 79675
Paper No 12 : Integral Transforms

Q. 1 Fill in the blanks (08 MCQ: 01 mark each)

1. L{ e^{at} . f(t) } = f(s – a) where L{f(t)}= f(s). This is ……

   A. change of scale property B. second shifting theorem

   C. effect of division D. first shifting theorem

2. If L{ f(t) } = f(s) then L{ f(t) / t} = …….

   A. $\int_s^\infty f(t)ds$ B. $\int_s^\infty f(s)dt$

   C. $\int_s^\infty f(s)ds$ D. $\int_s^\infty f(st)dt$

3. Laplace transform of e^{t^3} = …..

   A. $\frac{1}{s-3}$ B. $\frac{3}{s-1}$

   C. $\frac{1}{s+3}$ D. does not exist

4. L{ y'(t) } = …….

   A. sL{y}- sy(0) - y'(0) B. sL{y} + y(0)

   C. sL{y}- y(0) D. sL{y} + sy(0) + y'(0)

5. L{ $1/\sqrt{\pi t}$) } = ….

   A. $\frac{2}{\sqrt{s}}$ B. $\frac{1}{\sqrt{2s}}$

   C. $\frac{1}{\sqrt{s}}$ D. $\frac{1}{s}$

6. $\int_0^\infty \frac{sint}{t}\ dt$ = ….

   A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$

   C. $\frac{\pi}{6}$ D. $\frac{\pi}{3}$

7. $\int_0^{\infty} e^{-2t}\, t^2 \; dt$ = ….

A. $\frac{1}{4}$ B. $\frac{1}{2}$

C. $\frac{5}{6}$ D. $\frac{2}{3}$

8. L{ $\sin^2 t$} = ……..

A. $\frac{s}{s(s^2+4)}$ B. $\frac{4}{s(s^2+4)}$

C. $\frac{2}{s(s^2+4)}$ D. $\frac{4s}{s(s^2+4)}$

9. If f(t) is of class A then Laplace transform of f(t) exit. This is ….

A. Convolution theorem B. second shifting theorem

C. Existence theorem D. first shifting theorem

10. L{ t^n} = ……. if n is positive integer

A. $\frac{(2n)!}{s^{n+1}}$ B. $\frac{(n-1)!}{s^{n+1}}$

C. $\frac{(n+1)!}{s^{n+1}}$ D. $\frac{n!}{s^{n+1}}$

11. Laplace transform of $\int_0^t \int_0^t coshat\, dt\, dt$ = ……..

A. $\frac{s}{s^2-a^2}$ B. $\frac{1}{s(s^2-a^2)}$

C. $\frac{s}{s^2+a^2}$ D. $\frac{1}{s(s^2+a^2)}$

12. $L^{-1}\{\frac{2}{2S-1}\}$ = ……..

A. e^{2t} B. $e^{\frac{t}{2}}$

C. e^{4t} D. $e^{\frac{t}{4}}$

13. . $L^{-1}\{\ \frac{1}{s^2\,(s^2+1)}\ \}$ = ……..

A. t - cost B. t - sint

C. t - 2sint D. t + cost

14. $L^{-1}\{\ \frac{1}{s^3}\ \}$ = ……..

A. $\frac{t^2}{3!}$ B. $\frac{t^3}{4!}$

C. $\frac{t^3}{3!}$ D. $\frac{t^2}{2!}$

15. $L^{-1}\{$ f(s − a) $\}$ = …….. where L{f(t)}= f(s)

A. e^{at}f(at) B. e^{at}f(t)

C. e^{t}f(at) D. e^{at}f(2t)

16. L { 2^t} = ...

A. $\frac{1}{\mathrm{s-log2}}$ B. $\frac{1}{\mathrm{s-2}}$

C. $\frac{2}{\mathrm{s-log2}}$ D. does not exit

17. The conditions of Existence Theorem of Laplace transform are….

A. necessary B. sufficient but not necessary

C. necessary and sufficient D. necessary but not sufficient

18. $L^{-1}\{\ \frac{1}{\sqrt{s+2}}\ \}$ = ……..

A. $e^{-2t}\,\frac{1}{\sqrt{t}}$ B. $e^{-2t}\,\frac{1}{\sqrt{\pi t}}$

C. $e^{t}\,\frac{1}{\sqrt{\pi t}}$ D. $e^{-3t}\,\frac{1}{\sqrt{\pi t}}$

19. If $f_1(t) = \frac{1}{t}$, $f_2(t) = \frac{1}{\sqrt{t}}$ and s >0 then Laplace transform of

A. $f_1(t)$ does not exit but of $f_2(t)$ exits.

B. both $f_1(t)$ and $f_2(t)$ exit.

C. both $f_1(t)$ and $f_2(t)$ do not exit

D. $f_2(t)$ does not exit but of $f_1(t)$ exits.

20. Infinite Fourier transform of F(x) = 1 , $|x| < k$

= 0 , $|x| > k$

where F{F(x)} = $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x)e^{isx}\, dx$

A. $\sqrt{\frac{2}{\pi}}\ \frac{cossk}{s}$ B. $\sqrt{\frac{2}{\pi}}\ \frac{tansk}{s}$

C. $\sqrt{\frac{2}{\pi}}\ \frac{sinsk}{s}$ D. $\sqrt{\frac{2}{\pi}}\ \frac{sinsk}{k}$

21. The circular cosθ = …….

A. $\frac{e^{i\theta} - e^{-i\theta}}{2i}$ B. $\frac{e^{i\theta} - e^{-i\theta}}{2}$

C. $\frac{e^{i\theta} + e^{-i\theta}}{2i}$ D. $\frac{e^{i\theta} + e^{-i\theta}}{2}$

22. Infinite inverse Fourier sin transform of e^{-as} over 0 < s < ∞ is …

where F(x) = $\sqrt{\frac{2}{\pi}} \int_0^{\infty} f_s(s).sinsx\, ds$

A. $\sqrt{\frac{2}{\pi}} \cdot \frac{x}{(a^2 + x^2)}$ B. $\sqrt{\frac{2}{\pi}} \cdot \frac{x}{(a^2 - x^2)}$

C. $\sqrt{\frac{2}{\pi}} \cdot \frac{1}{(a^2 + x^2)}$ D. $\sqrt{\frac{2}{\pi}} \cdot \frac{1}{(a^2 - x^2)}$

23. If f(s) is Fourier transform of F(x) then Fourier transform of F(x).cosax is ….

A. ½ [f(s-a) - f(s+a)] B. ½ [f(s-a) + f(s+a)]

C. ½ [f(s) + f(s+a)] D. ½ [f(s-a) + f(s)]

24. If f (t) = 1 then Laplace transform of f(t) is …..

A. $\frac{1}{s}$ s<0 B. $\frac{1}{s}$ s>0

C. $\frac{1}{2s}$ s<0 D. $\frac{1}{2s}$ s>0

25. If L{ f(t) } = f(s) then L{ f(at) } = $\frac{1}{a}$f(s/a) . This is …….

A. change of scale property B. second shifting theorem

C. effect of division D. first shifting theorem

26. L { $e^{at}t^n$ } = …… s > a

A. $\frac{n!}{(s-a)^{n+1}}$ B. $\frac{n!}{(s+a)^{n+1}}$

C. $\frac{n}{(s-a)^{n+1}}$ D. $\frac{n!}{(s-a)^n}$

27. $\int_0^\infty e^{-3t}\, cos^2 t \;\; dt$ = ….

A. $\frac{11}{49}$ B. $\frac{10}{29}$

C. $\frac{11}{39}$ D. $\frac{12}{35}$

28. If L{ f(t) } = f(s) and G(t) = f(t-a) t > a

= 0 t < a

then L{ G(t) } =…..

A. e^{as}.f(s) B. e^{-as}.f(s)

C. e^{-s}.f(s) D. e^{-as}.f ’(s)

29. L{ $e^{-3t}.t^3$} = ……..

A. $\frac{4}{(s-3)^4}$ B. $\frac{3!}{(s-3)^4}$

C. $\frac{4!}{(s+3)^4}$ D. $\frac{3!}{(s+3)^4}$

30. L{ $(t+1)^2$ } = ……

A. $\frac{3}{s^3}+\frac{1}{s^2}+\frac{1}{s}$ B. $\frac{2}{s^3}+\frac{3}{s^2}+\frac{2}{s}$

C. $\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}$ D. $\frac{3}{s^3}+\frac{3}{s^2}+\frac{1}{s}$

31. L{ $\frac{1-e^{2t}}{t}$} = …….

A. - log $\frac{s-2}{s}$ B. log $\frac{s+2}{s}$

C. log $\frac{s}{s-2}$ D. log $\frac{s-2}{s}$

32. $L^{-1}\{\frac{1}{(s-3)^2}\}$ = ……..

A. $e^{2t}.t$ B. $e^{3t}.t$

C. $e^{3t}.t^2$ D. e^t.t

33. If $L\{$ f(t) $\}$ = f(s) then $L^{-1}\{f(as)\}$ =…..

A. $a\ L^{-1}\left\{f\left(\frac{s}{a}\right)\right\}$ B. 2a $L^{-1}\left\{f\left(\frac{s}{a}\right)\right\}$

C. $\frac{1}{a}\ L^{-1}\left\{f\left(\frac{s}{a}\right)\right\}$ D. $\frac{1}{a}\ L^{-1}\{f(s)\}$

34. $L^{-1}\{\ \frac{1}{(s^2+4)}\ \} = \ldots\ldots..$

A. $\frac{\sin 2t}{4}$ B. $\frac{\sin 4t}{2}$

C. $\frac{\sin 2t}{2}$ D. $\frac{\sin 4t}{4}$

35. $L^{-1}\{\ \frac{3s-7}{s^2-6s+8}\ \} = \ldots\ldots..$

A. e^{3t}[3cosht - 2sinht] B. e^{3t}[3cosht + 2sinht]

C. e^{3t}[3cost + 2sint] D. e^{3t}[3cosht + 2sint]

36. $L^{-1}\{\ \log(\frac{s+3}{s+2})\ \} = \ldots\ldots..$

A. $\frac{1}{t}\{\ e^{-3t} + e^{-2t}\ \}$ B. $\frac{-2}{t}\{\ e^{-3t} - e^{-2t}\ \}$

C. $\frac{2}{t}\{\ e^{-3t} + e^{-2t}\ \}$ D. $\frac{-1}{t}\{\ e^{-3t} - e^{-2t}\ \}$

37. Laplace transform of $(\sin t + \cos t)^2$ is …..

A. $\frac{s^2+4s+4}{s(s^2+4)}$ B. $\frac{s^2+2s+4}{s(s^2+4)}$

C. $\frac{s^2+2s+4}{s(s^2+16)}$ D. $\frac{s^2+4s+4}{s(s^2+16)}$

38. Infinite Fourier transform of $F(x) = \frac{1}{2\epsilon}$, $|x| < \epsilon$

$= 0$, $|x| > \epsilon$ is…

where $F\{F(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x)e^{isx}\,dx$

A. $\frac{1}{\sqrt{2\pi}}\ \frac{\cos s\epsilon}{s}$ B. $\frac{1}{\sqrt{2\pi}}\ \frac{\tan s\epsilon}{s}$

C. $\frac{1}{\sqrt{2\pi}}\ \frac{\sin s\epsilon}{s\epsilon}$ D. $\frac{1}{\sqrt{2\pi}}\ \frac{\sin s\epsilon}{\epsilon}$

39. Infinite Fourier sine transform of F(x) = $\frac{1}{x}$ over $0 < x < \infty$ is ……

where $f_s(s) = \sqrt{\frac{2}{\pi}} \int_0^\infty F(x) \sin sx \, dx$

A. $\sqrt{\frac{\pi}{4}}$ B. $\sqrt{\frac{2}{\pi}}$

C. $\sqrt{\frac{\pi}{2}}$ D. $\sqrt{\frac{3}{\pi}}$

40. IF F (t) = e^{-xt} . ∅(t) , t > 0

= 0, t < 0

where $F\{F(t)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(t) e^{iyt} \, dt$

Then the relation between Laplace and Fourier transform

for this function is ……

A. $F\{F(t)\} = \frac{1}{\sqrt{2\pi}} L\{ \emptyset(t) \}$ B. $F\{F(t)\} = \frac{1}{\sqrt{2\pi}} L\{ \emptyset'(t) \}$

C. $F\{F'(t)\} = \frac{1}{\sqrt{2\pi}} L\{ \emptyset(t) \}$ D. $F\{F(t)\} = \frac{t}{\sqrt{2\pi}} L\{ \emptyset(t) \}$

Q.2 Attempt any Two (08 mark each)

1) State and prove Existence theorem of Laplace transform.

2) If L{f(t)} = f(s) then prove that a) $L\{ e^{at} . f(t) \} = f(s - a)$

b) $L\{ e^{-at} . f(t) \} = f(s + a)$

3) If L{f(t)} = f(s) then prove that $L\{ f'(t) \} = sf(s) - f(0)$

4) If L{f(t)} = f(s) then prove that $L\{\int_0^t f(u)du \} = \frac{1}{s} f(s)$

5) If L{f(t)} = f(s) then prove that $L\{ t^n f(t) \} = (-1)^n \frac{d^n}{ds^n} [f(s)]$,n = 1,2,3…

6) If f (t) is periodic function of period T then prove that

$L\{ f(t) \} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt$

7) If $L^{-1}\{f(s)\} = f(t)$ then prove that $L^{-1}\{ \int_s^\infty f(s) ds \} = \frac{f(t)}{t}$

8) If $L\{F(t)\} = f(s)$ and $L\{G(t)\} = g(s)$ then prove the Convolution theorem

$L^{-1}\{f(s).\, g(s)\} = F(t)*G(t) = \int_0^t F(u) G(t-u)\} du$

9) By Convolution theorem find inverse Laplace transform of $\frac{1}{s(s^2+2s+2)}$

10) By Convolution theorem find inverse Laplace transform of $\frac{1}{(s+a)(s+b)}$

11) IF $F(t) = e^{-xt} . \emptyset(t)$, $t > 0$

$= 0$, $t < 0$ where $F\{F(t)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(t) e^{iyt}\, dt$

Then prove that $F\{F(t)\} = \frac{1}{\sqrt{2\pi}} L\{ \emptyset(t) \}$

12) Obtain infinite Fourier sine transform of $F(x) = \frac{1}{x}$ over $0 < x < \infty$

13) Obtain infinite Fourier transform of $F(x) = \frac{1}{2\epsilon}$, $|x| < \epsilon$

$= 0$, $|x| > \epsilon$

where $F\{F(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x) e^{isx}\, dx$

14) If f(s) is Fourier transform of F(x) then prove that Fourier transform of

$F(x).\cos ax$ is $\frac{1}{2}[f(s - a) + f(s + a)]$

15) Obtain infinite Fourier transform of $F(x) = 1$, $|x| < k$

$= 0$, $|x| > k$

where $F\{F(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x) e^{isx}\, dx$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$

Q.3 Attempt any Four (04 marks each)

Find the Laplace transform of the following functions.

1 $f(t) = \sin(2t)\cos(2t)$

2 $f(t) = \cos^2(3t)$

3 $f(t) = te^{2t}\sin(3t)$

4 $f(t) = (\text{sint} + \text{cost})^2$

5 $f(t) = (\text{sint} - \text{cost})^2$

6 $f(t) = \begin{cases} t, & \text{if} \quad 0 \le t < 3, \\ 5, & \text{if} \quad t \ge 3 \end{cases}$

7 $f(t) = \frac{sint}{t}$

8 $f(t) = \frac{sinht}{t}$

Evaluate the following integrals

9 Evaluate $\int_0^\infty e^{-t}\, t^3 \; dt$ using Laplace transform.

10 Evaluate $\int_0^\infty e^{-3t}\, t \; dt$ using Laplace transform.

Find the inverse Laplace Transform:

11 $f(s) = \frac{1}{(s+1)(s-1)}$

12 $f(s) = \frac{2s+3}{s^2+4s+13}$

13 $f(s) = \frac{2s+1}{s^2-4}$

14 $f(s) = \frac{s+3}{s^2-10s+29}$

15 Find the Fourier transform of $\text{F(x)} = \begin{cases} x, & |x| < a \\ 0, & |x| > a \end{cases}$.

15 Find the Fourier transform of f(x) $= \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$

16 Find the Fourier transform of $f(x) = x.e^{-x}$ 0< x <∞

17 Find the infinite Fourier sine transform of $f(x) = x$ if 0< x <∞

18 Find the infinite Fourier cosine transform of $f(x) = x$ if 0< x <∞

19 Find the infinite Fourier sine transform of $f(x) = e^{-ax}$ if 0< x <∞

20 Find the infinite Fourier cosine transform of $f(x) = e^{-ax}$, if $x > 0$.

21 Find the finite Fourier sine transform of f(x) = 2x 0<x<4

22 Find the finite Fourier cosine transform of f(x) = 2x 0<x<4

23 Find the infinite Fourier cosine transform of $f(x) =$

$$\begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$

24 Find the infinite Fourier cosine transform of e^{-x^2} 0< x <∞

26 Prove that $L\{e^{at}\} = \frac{1}{s-a}$

27 Prove that $L\{\sinh at\} = \frac{a}{s^2-a^2}$

28 Find inverses Laplace transform of log $(\frac{s+3}{s+2})$

29 Find inverse Laplace transform of log $(\frac{s+3}{s+1})$

30 Find Laplace transform of $\frac{1}{\sqrt{t}}$