Revised Syllabus for **B. Sc. Part III [Mathematics] (Sem. V & VI)** to be implemented from June 2015.

1. TITLE: **Subject : Mathematics**
2. YEAR OF IMPLEMENTATION: Revised Syllabus will be implemented from June 2015 onwards.

3.DURATION: B.Sc. Part - III - The duration of course shall be one year and two semesters.

4. PATTERN: Pattern of examination will be semester

5. MEDIUM OF INSTRUCTION: English.

6. STRUCTURE OF COURSE: THIRD YEAR B. Sc. (MATHEMATICS)
(Semester V & VI)

**Semester V**

| **Sr.No.** | **Paper No.** | **Name of the Paper** | **Marks** | |
|---|---|---|---|---|
| | | | **Theory** | **Internal** |
| 1 | IX | Real Analysis | 40 | 10 |
| 2 | X | Modern Algebra | 40 | 10 |
| 3 | XI | Partial Differential Equations | 40 | 10 |
| 4 | XII | Numerical Methods -I | 40 | 10 |

**Semester VI**

| **Sr.No.** | **Paper No.** | **Name of the Paper** | **Marks** | |
|---|---|---|---|---|
| | | | **Theory** | **Internal** |
| 1 | XIII | Metric Spaces | 40 | 10 |
| 2 | XIV | Linear Algebra | 40 | 10 |
| 3 | XV | Complex Analysis | 40 | 10 |
| 4 | XVI | Numerical Methods- II | 40 | 10 |

## Practical (Annual Pattern)

| | |
|---|---|
| Computational Mathematics Laboratory – IV<br>(Operations Research Techniques) | 50 Marks |
| Computational Mathematics Laboratory –V<br>(Numerical Methods) | 50 Marks |
| Computational Mathematics Laboratory – V<br>Numerical Recipes in C++, SciLab ) | 50 Marks |
| Computational Mathematics Laboratory – VII<br>(Project Work, Study Tour, Viva - Voce) | 50 Marks |

**EQIVALENCE IN ACCORDANCE WITH TITLES AND CONTENTS OF PAPERS (FOR REVISED SYLLABUS**

| Sr.No. | Title of Old Paper | | Title of New Paper | |
|---|---|---|---|---|
| | **Paper No.** | **Name of the Paper** | **Paper No.** | **Name of the Paper** |
| 1 | IX | Real Analysis | IX | Real Analysis |
| 2 | X | Modern Algebra | X | Modern Algebra |
| 3 | XI | Partial Differential Equations | XI | Partial Differential Equations |
| 4 | XII | Numerical Methods -I | XII(A) | Symbolic Logic & Graph Theory |
| | | | XII(B) | Special Theory of Relativity – I |
| | | | XII(C) | Differential Geometry – I |
| | | | XII(D) | Mathematical Modeling -I |
| | | | XII(E) | Applications of Mathematics in Finance |
| | | | XII(F) | Mechanics – I |
| 5 | XIII | Metric Spaces | XIII | Metric Spaces |
| 6 | XIV | Linear Algebra | XIV | Linear Algebra |
| 7 | XV | Complex Analysis | XV | Complex Analysis |
| 8 | XVI | Numerical Methods-II | XVI(A) | Algorithms & Boolean Algebra |
| | | | XVI (B) | Special Theory of Relativity – II |
| | | | XVI (C) | Differential Geometry – II |
| | | | XVI (D) | Mathematical Modeling -II |
| | | | XVI (E) | Applications of Mathematics in Insurance |
| | | | XVI (F) | Mechanics – I |

**EQIVALENCE IN ACCORDANCE WITH TITLES AND CONTENTS OF COMPUTATIONAL MATHEMATICS LABORATRY (FOR REVISED SYLLABUS**

| Sr.No. | Title of Old Paper | Title of New Paper |
|---|---|---|
| 1 | Computational Mathematics Laboratory-IV (Operations Research Techniques) | Computational Mathematics Laboratory- IV (Operations Research Techniques) |
| 2 | Computational Mathematics Laboratory -V (Laplace Transform) | Computational Mathematics Laboratory -V (Numerical Methods) |
| 3 | Computational Mathematics Laboratory -VI (Numerical Recipes in C++, MATHLAB & Microsoft Excel) | Computational Mathematics Laboratory -VI (Numerical Recipes in C++, SciLab) |
| 4 | Computational Mathematics Laboratory-VII (Project Work, Study Tour, Viva - Voce) | Computational Mathematics Laboratory-VII (Project Work, Study Tour, Viva - Voce) |

# SHIVAJI UNIVERSITY, KOLHAPUR
## B.Sc. Part-III (Mathematics)
# Detail syllabus of semester V and VI

## SEMESTER – V
## Paper – IX
## REAL ANALYSIS

UNIT – 1 **SETS AND FUNCTIONS** 5 lectures

**1.1 Sets and Elements, Operations on sets**

**1.2 Functions**

1.2.1 Definition of Cartesian product, Function, Extension and restriction of functions, onto function.

1.2.2 THEOREM: If $f: A \to B$ and if $X \subset B, Y \subset B$, then

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y).$$

1.2.3 THEOREM: If $f: A \to B$ and if $X \subset B, Y \subset B$, then

$$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y).$$

1.2.4 THEOREM: If $f: A \to B$ and if $X \subset A, Y \subset A$, then

$$f(X \cup Y) = f(X) \cup f(Y).$$

12.5 THEOREM: If $f: A \to B$ and if, $X \subset A, Y \subset A$, then

$$f(X \cap Y) \subset f(X) \cap f(Y).$$

1.2.6 Definition of composition of functions.

**1.3 Real-valued functions**

1.3.1 Definition : Real valued function. Sum, difference, product, and Quotient of real valued functions, $\max(f.g)$, $\min(f,g)$, $|f|$, Characteristic Function.

**1.4 Equivalence, Countability**

1.4.1 Definitions: one – to – one function, inverse function, 1–1 correspondence and equivalent sets, finite and infinite sets, countable and uncountable set.

1.4.2 Theorem: The countable union of countable sets is countable.

1.4.3 Corollary: The set of rational numbers is countable.

1.4.4 Theorem: If B is an infinite subset of the countable set A, then B is countable.

1.4.5 Corollary: The set of all rational numbers in $[0,1]$ is countable.

**1.5 Real numbers**

1.5.1 Theorem: The set $[0,1] = \{x : 0 \le x \le 1\}$ is uncountable.

1.5.2 Corollary: The set of all real numbers is uncountable.

**1.6 Least upper bounds**

1.6.1 Definition: Upper bound, lower bound of a set, least upper bound.

1.6.2 Least upper bound axiom,

1.6.3 Theorem: If A is any non-empty subset of R that is bounded below, then A has a greatest lower bound in R.

UNIT – 2 **SEQUENCES AND SERIES OF REAL NUMBERS** 20 lectures

**2.1 Definition of sequence and subsequence**

**2.2 Limit of a sequence**

2.2.1 Definition.

2.2.2 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of nonnegative numbers and if $\lim_{n \to \infty} s_n = L$, then $L \ge 0$.

**2.3 Convergent sequence**

2.3.1 Definition

2.3.2 Theorem: If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then cannot also converge to a limit distinct from L. That is, if $\lim_{n \to \infty} s_n = L$ and $\lim_{n \to \infty} s_n = M$, then $L = M$.

2.3.3 Theorem: If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent to L, then any subsequence of $\{s_n\}_{n=1}^{\infty}$ is also convergent to L.

2.3.4 Theorem: All subsequences of real numbers converge to same limit.

## 2.4 Divergent sequences

2.4.1 Definitions.

## 2.5 Bounded sequences

2.5.1 Definition.

2.5.2 Theorem: If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then $\{s_n\}_{n=1}^{\infty}$ is bounded.

## 2.6 Monotone sequences

2.6.1 Definition.

2.6.2 Theorem: A non decreasing sequence which is bounded above is convergent.

2.6.3 Theorem: The sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent.

2.6.4 Theorem: A non decreasing sequence which is not bounded above diverges to infinity.

2.6.5 Theorem: A non increasing sequence which is bounded below is convergent.

2.6.6 Theorem: A non increasing sequence which is not bounded below diverges to minus infinity.

## 2.7 Operations on convergent sequences

2.7.1 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n$ = L and $\lim_{n\to\infty} t_n$ = M, then $\lim_{n\to\infty}(s_n + t_n)$ = L + M.

In words the limit of sum (of two convergent sequences) is the sum of the limits.

2.7.2 Theorem: $\{S_n\}_{n=1}^{\infty}$ is of real numbers, if c $\in$ R, and if

$\lim_{n\to\infty} S_n = L$ then $\lim_{n\to\infty} cS_n = cL$.

2.7.3 Theorem: (a) If 0 < x < 1, then $\{x^n\}_{n=1}^{\infty}$ converges to 0.

(b) If 1< x <$\infty$, then $\{x^n\}_{n=1}^{\infty}$ diverges to infinity.

2.7.4 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} t_n = M$, then $\lim_{n\to\infty}(S_n - t_n) = L - M$.

2.7.5 Theorem: If $\{S_n\}_{n=1}^{\infty}$ is sequence of real numbers which converges to L , then $\{S^2{}_n\}_{n=1}^{\infty}$ converges to $L^2$.

2.7.6 Theorem: If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} t_n = M$, then $\lim_{n\to\infty}(S_n . t_n) = LM$.

2.7.7 Theorem: Theorem: If $\{t_n\}_{n=1}^{\infty}$ is sequence of real numbers and if $\lim_{n\to\infty} t_n = M$, where $M \neq 0$ then $\lim_{n\to\infty} \frac{1}{t_n} = \frac{1}{M}$.

2.7.8 Theorem: $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} t_n = M$, then $\lim_{n\to\infty}\left({S_n}/{t_n}\right) = L/M$.

**2.2 Limit superior and limit inferior**

2.2.1 Definition: Limit superior and limit inferior and Examples.

2.2.2 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then

$$\limsup_{n\to\infty} s_n = \lim_{n\to\infty} s_n.$$

2.2.3 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then

$$\liminf_{n\to\infty} s_n = \lim_{n\to\infty} s_n.$$

2.2.4 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, then

$$\limsup_{n\to\infty} s_n \geq \liminf_{n\to\infty} s_n.$$

2.2.5 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, and if $\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = L$ and $L \in R$, then $\{s_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} s_n = L$.

2.2.6 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, and if $\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = \infty$, then $\{s_n\}_{n=1}^{\infty}$ diverges to infinity.

2.2.7 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers, and if $s_n \le t_n \quad (n \in I)$, then $\limsup_{n\to\infty} s_n \le \limsup_{n\to\infty} t_n$ and $\liminf_{n\to\infty} s_n \le \liminf_{n\to\infty} t_n$.

2.2.8 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real Numbers, then

$$\limsup_{n\to\infty}(s_n + t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n \ ;$$

$$\liminf_{n\to\infty}(s_n + t_n) \ge \liminf_{n\to\infty} s_n + \liminf_{n\to\infty} t_n \, .$$

2.2.9 Theorem ( Statement only): Let $\{s_n\}_{n=1}^{\infty}$ be bounded sequences of real Numbers.

a) If $\limsup_{n\to\infty} s_n = M$, then for any $\varepsilon > 0$, (a) $s_n < M + \varepsilon$ for all but a finite number of values of n; (b) $s_n > M - \varepsilon$ for infinitely many values of n.

b) If $\liminf_{n\to\infty} s_n = m$, then for any $\varepsilon > 0$, (c) $s_n > m - \varepsilon$ for all but a finite number of values of n; (d) $s_n < m + \varepsilon$ for infinitely many values of n.

2.2.10 Theorem: Any bounded sequence of real numbers has a convergent subsequence.

## 2.3 Cauchy sequences ( Revision and statements of standard results without Proof)

## 2.4 Summability of sequences

2.4.1 Definition: (C,1) summablility and examples.

2.4.2 Theorem: If $\lim_{n\to\infty} s_n = L$, then $\lim_{n\to\infty} s_n = L$ (C,1).

## 2.5 Series of Real numbers

2.5.1 **Definition : Series Convergence and divergence.**

2.5.2 Theorem : If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then $\sum_{n=1}^{\infty} (a_n + b_n)$ converges to A + B.

2.5.3 Theorem : If $\sum_{n=1}^{\infty} a_n$ converges to A and $\sum_{n=1}^{\infty} b_n$ converges to B, then $\sum_{n=1}^{\infty} (a_n - b_n)$ converges to A – B.

2.5.4 Theorem : If $\sum_{n=1}^{\infty} a_n$ converges to A and c $\in$ R, then $\sum_{n=1}^{\infty} c\, a_n$ converges to cA.

2.5.5 Theorem : If $\sum_{n=1}^{\infty} a_n$ is convergent sequence, then $\lim_{n\to\infty} a_n = 0$.

## 2.6 Series whose terms form a non-increasing sequence

2.6.1 Theorem: If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=0}^{\infty} 2^n a_{2^n}$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. Examples.

2.6.2 Theorem: If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=0}^{\infty} 2^n a_{2^n}$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges. Examples.

2.6.3 Theorem: The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

2.6.4 Theorem: If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} na_n = 0$. Examples.

## 2.7 Summation by parts

2.7.1 Theorem: If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are two sequences of real numbers and let $s_n = a_1 + \cdots + a_n$. Then, for each positive integer n ∈ I,

$$\sum_{k=1}^{n} a_k b_k = s_n b_{n+1} - \sum_{k=1}^{n} s_k (b_{k+1} - b_n).$$

2.7.2 Abel′s lemma

2.7.3 Dirichlet′s test

2.7.4 Abel′s test

2.7.5 Examples

**2.8 (C,1) Summability of series**

2.8.1 Definition of (C,1) Summability of series.,

2.8.2 Theorem: If $\sum_{n=1}^{\infty} a_n$ is (C,1) summable and if $\lim_{n\to\infty} na_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

**2.9 The Class** $l^2$

2.9.1 Definition of the class $l^2$.

2.9.2 Theorem : The Schwarz inequality.

2.9.3 Theorem : Minkowski inequality.

2.9.4 Norm of an element in $l^2$.

**2.9.5 Theorem:** The norm for sequences in $l^2$ has the following properties:

$N_1$: $\|s\|_2 \geq 0$ $(s \in l^2)$,

$N_2$: $\|s\|_2 = 0$ if and only if $s = \{0\}_{n=1}^{\infty}$,

$N_3$: $\|cs\|_2 = |c|.\|s\|_2$ $(c \in R,\ s \in l^2)$,

$N_4$: $\|s + t\|_2 \leq \|s\|_2 + \|t\|_2$ $(s, t \in l^2)$.

**UNIT –3 : <u>RIEMANN INTEGRATION</u>** **11 lectures**

**3.1** Riemann integrability & integrals of bounded functions over bounded intervals:

3.1.1 **Definitions & simple examples:** subdivision & norm of subdivision, lower & upper sums, lower & upper integrals, oscillatory sum, Riemann integral.

**3.2 Darboux's Theorem:**

**3.2.1 Lemma:** Let f(x) be a function defined on [a, b] for which there is a $k \in R$ such that $|f(x)| \leq k$. Let $D_1$ be a subdivision of [a, b] and $D_2$ be the subdivision of [a, b] consisting of all points of $D_1$ and at the most p more, with $|D_1| \leq \delta$. Then $S(D_1) - 2pk\delta \leq S(D_2) \leq S(D_1)$

**3.2.2 Theorem:** To every $\in > 0$, there corresponds $\delta > 0$ such that $S(D) < \int_a^b f(x)\,dx + \in$, for every D with $|D| \leq \delta$.

**3.2.3 Theorem:** To every $\in > 0$, there corresponds $\delta > 0$ such that $s(D) > \int_a^b f(x)\,dx - \in$, for every D with $|D| \leq \delta$ .

**3.2.4 Theorem:** For every bounded function f on [a, b], prove that the upper integral $\geq$ the lower integral.

### 3.3 Equivalent definition of integrability and integrals.

**3.3.1 Theorem:** If f is bounded and integrable over [a, b], then to $\varepsilon > 0$ there corresponds $\delta > 0$, such that for every subdivision $D = \{a = x_0, x_1, x_2, \ldots, x_n = b\}$ with $|D| \leq \delta$ and for every choice of $\xi_r \in [x_{r-1}, x_r]$, $\left|\sum_{r=1}^{n} f(\xi_r)(x_r - x_{r-1}) - \int_a^b f(x)dx\right| < \varepsilon$.

**3.3.2 Theorem:** If f is integrable over [a, b] and if there exists a number I such that to every $\varepsilon > 0$ there correspond $\delta > 0$ such that for every subdivision $D = \{a = x_0, x_1, x_2, \ldots, x_n = b\}$ with $|D| \leq \delta$ and for every choice of $\xi_r \in [x_{r-1}, x_r]$ with $|\sum_{r=1}^{n} f(\xi_r)(x_r - x_{r-1}) - I| < \varepsilon$ then I is value of $\int_a^b f(x)dx$.

### 3.4 Conditions for integrability.

**3.4.1 Theorem:** The necessary and sufficient condition for the integrability of a bounded function f over [a, b] is that to every $\varepsilon > 0$, there corresponds $\delta > 0$ such that for every subdivision D of [a, b] with $|D| \leq \delta$, the oscillatory sum $w(D) < \varepsilon$.

**3.4.2 Theorem:** The necessary and sufficient condition for the integrability of a bounded function f over [a, b] is that to every $\varepsilon > 0$, there corresponds a subdivision D of [a, b] such that the corresponding oscillatory sum $w(D) < \varepsilon$.

### 3.5 Particular classes of bounded integrable functions:

**3.5.1 Theorem:** Every continuous function on [a, b] is Riemann

**3.5.2 Theorem:** Every monotonic function on [a, b] is Riemann integrable.

**3.5.3 Theorem:** Every bounded function on [a, b] which has only a finite number of points of discontinuities is Riemann integrable.

**3.5.4 Theorem:** If the function f bounded on [a, b] and the set of all points of discontinuities has a finite number of limit points then f is Riemann integrable over [a, b].

**3.5.5 Examples on 3.5.**

**3.6 Properties of integrable functions**:

**3.6.1 Theorem:** If a bounded function f is integrable on [a, b] then f is also integrable on [a, c] & [c, b], for a < c < b and conversely. In this case $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.

**3.6.2 Lemma:** The oscillation of a bounded function f on [a, b] is the least upper bound of the set $\{|f(\alpha)-f(\beta)|/\alpha, \beta \in [a,b]\}$.

**3.6.3 Theorem:** If f & g are both bounded and integrable functions on [a, b] then f ± g are also bounded & integrable over [a, b] and $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$.

**3.6.4 Theorem:** If f & g are both bounded and integrable functions on [a, b ] then the product f.g is also bounded & integrable over [a, b].

**3.6.5 Theorem:** If f & g are both bounded and integrable functions on [a, b] and if there exists t > 0 with |g(x)| ≥ t, (a ≤ x ≤ b) then $f/g$ is also bounded & integrable over [a, b].

**3.6.6 Theorem:** If f is both bounded and integrable function on [a, b] then | f | is also bounded & integrable over [a, b].

**3.7 Inequalities for an integral:**

**3.7.1 Theorem:** If f is bounded and integrable function on [a, b] and if M and m are the least upper and greatest lower bounds of f over [a, b] then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$, if a ≤ b and

$$m(a-b) \geq \int_a^b f(x)dx \geq M(a-b), \text{ if } b \leq a.$$

**3.7.2 Theorem:** If f is bounded and integrable over [a, b] with |f(x)| ≤ k then $\left|\int_a^b f(x)dx\right| \leq k.|b-a|$.

**3.7.3 Theorem:** If $\int_a^b |f(x)|dx$ exists then $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$.

**3.8 Function defined by a definite integral:**

**3.8.1 Definition:** the integral function of an integrable function f on [a, b].

**3.8.2 Theorem:** The integral function of an integrable function is continuous.

**3.8.3 Theorem:** The integral function Φ, of a continuous function f, is continuous and $\Phi' = f$.

**3.9 Theorems of Integral Calculus (statements only):**

3.9.1 Fundamental Theorem of Integral calculus.

3.9.2 First Mean Value Theorem of Integral calculus.

3.9.3 Second Mean Value Theorem of Integral calculus.

3.9.4 Integration by Change of variable.

3.9.5 Integration by Parts.

**3.10 Examples on 3.9.**

**UNIT – 4 : <u>IMPROPER INTEGRALS</u>** **9 lectures**

**4.1 Definitions & simple examples:** a point of infinite discontinuity of a function, a proper integral, an improper integral, convergence at the left end, convergence at the right end, convergence at both the ends, the case of finite number of infinite discontinuities, convergence at infinity convergence at minus infinity, convergence over (∞, – ∞), convergence over (∞, – ∞) together with a finite number of infinite discontinuities.

**4. 2 Test for convergence at the left end: positive integrand.**

4.2.1 The necessary & sufficient condition for the convergence of the improper integral $\int_a^b f(x)dx$ at a, when f(x)>0 for all x Є (a , b].

4.2.2 Comparison test for two improper integrals & examples.

4.2.3 Practical comparison test for two improper integrals & examples.

4.2.4 A useful comparison integral $\int_a^b \frac{dx}{(x-a)^n}$ & examples.

4.2.5 Examples on 4.2.

**4.3 General test for convergence of the improper integral $\int_a^b f(x)dx$ at a ; f(x), not necessarily positive:**

**4.3.1** Definition of absolute convergence of the improper integral $\int_a^b f(x)dx$ at a.

**4.3.2 Theorem:** Every absolutely convergent integral is convergent.

**4.3.3** Convergence at ∞ of the improper integral $\int_a^\infty f(x)dx$: positive integrand .

**4.3.4** A useful comparison integral $\int_a^\infty \frac{dx}{x^n}$ ,a>0. The necessary & sufficient condition for its convergence.

**4.3.5 Examples on 4.3.**

**4.4** Convergence at ∞ , the integrand being not necessarily positive.

**4.4.1** General test for convergence.

**4.4.2 Theorem:** If an improper integral converges absolutely then it converges.

**4.4.3** Test for absolute convergence of the integral of a product.

**4.4.4 Examples on 4.4.**

**4.5 Tests for conditional convergence:**

**4.5.1** Abel's theorem for the convergence of the integral of a product.

**4.5.2** Dirichlet's theorem for the convergence of the integral of a product.

**4.5.3 Examples on 4.5.**

## REFERENCE BOOKS


1. R. R. Goldberg, **Methods of Real Analysis,** Oxford and IBH Publishing House.

**2.** T. M. Apostol, **Mathematical Analysis,** Narosa Publishing House

**3.** Satish Shirali, H. L. Vasudeva, **Mathematical Analysis,** Narosa Publishing House.

**4.** D. Somasundaram, B. Choudhary, **First Course in Mathematical Analysis,** Narosa Publishing House

**5.** W. Rudin, **Principles of Mathematical Analysis,** McGraw Hill Book Company.

**6**. Shantinarayan, Mittal, **A Course of Mathematical Analysis,** S.Chand and Company.

**7.** J.N. Sharma, **Mathematical Analysis-**I, Krishna Prakashan Mandir, Meerut.

**8.** Malik, Arrora, **Mathematical Analysis,** Wiley Eastern Ltd.

**9.** Shantinarayan, M.D.Raisinghania, **Mathematical Analysis,** S.Chand

# Paper – X
# (Modern Algebra)

**Unit – 1 GROUPS** **13 lectures**

**1.1** Divisibility in integers, the greatest common divisor (g.c.d.), function from a set A to a set B, a 1-1 function, an onto function, prime and composite numbers, congruence relation on the set of integers, Permutations, Cyclic Permutations, Transpositions, Disjoint Permutations, Even and odd permutations.

**1.1.1 Theorem:** (without proof): Euclid's algorithm: If m is a positive integer and n is any integer then there exists integers q and r such that $n = mq + r$, where $0 \le r < m$.

**1.1.2 Theorem:** (without proof): For any two non-zero integers a,b there exists a greatest common divisor d of a and b such that $d = ax + by$ for some integers x and y.

**1.1.3Theorem:** (without proof): Two integers a and b are relatively prime if and only if there exist two integers x & y such that $ax + by = 1$.

**1.2** Definitions & examples of Groups, commutative group**,** Order of a group, Quaternion group, Group of residues.

**1.2.1 Theorem**s: In a group G,

a) Identity element is unique.

b) Inverses of each $a \in G$ is unique.

c) $(a^{-1})^{-1} = a$, for all $a \in G$.

d) $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

e) $ab = ac \Rightarrow b = c$, $ba = ca \Rightarrow b = c$ for all $a, b, c \in G$.

**1.2.2 Theorem :** For elements a, b in a group G, the equations $ax = b$ and $ya = b$ have unique solutions for x and y in G.

**1.3 Definition of Subgroup and examples.**

**1.3.1 Theorem:** A non empty subset H of a group G is subgroup of G if and only if i) $a, b \in H \Rightarrow ab \in H$, ii) $a \in H \Rightarrow a^{-1} \in H$.

**1.3.2 Theorem:** A non-empty subset H of a group G is a subgroup of G if and only if $a, b \in H \Rightarrow ab^{-1} \in H$

**1.3.3 Theorem:** A non empty finite subset H of a group G is a sub group of G if and only if H is closed under multiplication.

**1.4 Definition of Centre of group G, Normalizer of an element in G**.

**1.4.1 Theorem**: Centre of group G is a subgroup of G

**1.4.2** Normalizer is a subgroup of G.

**1.4.3** Union and intersection of subgroups

**1.5** Definition of left and right cosets and congruence relation.

**1.5.1 Theorem : (i)** $Ha = H$ if and only if $a \in H$. (ii) $Ha = Hb$ if and only if $ab^{-1} \in H$ .(iii) Ha is a subgroup of G if and only if $a \in H$.

**1.5.2 Theorem**: $Ha = \{x \in G \mid x \equiv a \bmod H\} = cl(a)$ for any $a \in G$.

**1.5.3 Lemma**: Any two right cosets have same number of elements.

**1.5.4 Lagrange's Theorem**: If G is a finite group and H is a subgroup of G then o(H) divides o(G).

**1.6** Index of H in G, Centralizer of H, Normalizer of H

**1.6.1 Theorem**: HK is subgroup of G if and only if HK = KH.

**1.7** Definition of Cyclic group and Order of element of a group.

**1.7.1 Theorem**: Order of a cyclic group is equal to the order of its generator.

**1.7.2 Theorem**: A subgroup of a cyclic group is cyclic.

**1.7.3 Theorem**: A cyclic group is abelian.

**1.7.4 Theorem**: If G is finite group, then order of any element of G divides order of group G.

**1.7.5 Theorem**: An infinite cyclic group has precisely two generators.

**1.7.6** Definition of Eule's $\phi$ function.

**1.7.7 Theorem**: Number of generators of a finite cyclic group of order n is $\phi(n)$.

**1.7.8 Euler's Theorem** : Let a, n (n $\geq 1$) be any integers such that g.c.d.(a,n) =1, then $a^{\phi(n)} \equiv 1 \pmod{n}$.

**1.7.9 Fermat's Theorem**: For any integer a and prime p, $a^p \equiv a \pmod{p}$

**1.7.10** Examples.

**Unit 2 NORMAL SUBGROUPS, HOMOMORPHISM, PERMUTATION GROUP 12 Lectures**

**2.1** Definition and examples of normal Subgroup, simple group, quotient group

**2.1.1 Theorem**: A subgroup H of a group G is normal in G if and only if $g^{-1}Hg = H$, $g \in G$.

**2.1.2 Theorem**: A subgroup H of a group G is normal in G if and only if $g^{-1}hg \in H$ for all $h \in H$, $g \in G$.

**2.1.3 Theorem**: A subgroup H of a group G is normal in G if and only if the product of two right (left) cosets of H in G is again a right (left) coset of H in G.

**2.1.4** The normaliser N(H).

**2.1.5 Theorem**: If G is finite group and N is a normal subgroup of G then

$$o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}$$

**2.1.6 Theorem**: Every quotient group of a cyclic group is cyclic

**2.2** Definition and examples of Homomorphism, Isomorphism, Epimorphism, Monomorphism, Endomorphism and Automorphism.

**2.2.1 Theorem** : If $f : G \to G'$ is homomorphism then

(i) $f(e) = e'$ (ii) $f(x^{-1}) = [f(x)]^{-1}$

(iii) $f(x^n) = [f(x)]^n$ , n is an integer.

**2.2.3** Definition of Kernel of homomorphism.

**2.2.4 Theorem**: If $f : G \to G'$ is homorphism then Ker f is a normal subgroup of G.

**2.2.5 Theorem**: A homomorphism $f : G \to G'$ is one-one if and only if Ker f = {e}.

**2.2.6 Fundamental Theorem of group homomorphism**: If $f: G \rightarrow G'$ is an onto homomorphism with K = Ker f, then $\frac{G}{K} \cong G'$

**2.2.7 Second Theorem of isomorphism**: Let H and K be two subgroups of group G, where H is normal in G, then $\frac{HK}{H} \cong \frac{K}{H \cap K}$

**2.2.8 Third Theorem of isomorphism**: If H and K be two normal subgroups of group G, such that H $\subseteq$ K then $\frac{G}{K} \cong \frac{G/H}{K/H}$

**2.2.9** Dihedral group, Permutation group.

**2.2.10 Cayley's Theorem**: Every group G is isomorphic to a permutation group.

**2.2.11** Set of even permutations is a normal subgroup of $S_n$ .Alternating group.

**Unit –3 RINGS** **10 Lectures**

**3.1** Definition and examples of a Ring, Commutative ring, Ring with unity.

**3.1.1** Theorem: In a ring R (i) a.0 = 0.a = 0, (ii) a(– b) = (– a)b = – ab, (iii)( – a)( – b) = ab, (iv) a(b – c) = ab – ac.

**3.1.2** Zero divisor, Integral Domain, Division Ring, Field,

**3.1.3 Theorem**: A commutative ring R is an integral domain iff cancellation law holds.

**3.1.4 Theorem** (i) A field is an integral domain. (ii) A non-zero finite integral domain is a field.

**3.1.5 Boolean ring :** Every Boolean ring is (i) commutative (ii) of the order $2^n$

**3.2** Definition and examples of Subring

**3.2.1Theorem**: A non-empty subset S of ring R is a subring of R iff a, b $\in$ S $\Rightarrow$ ab, a – b $\in$ S.

**3.3** Characteristic of a ring: Definition and examples.

**3.3.1 Theorem**: Let R be ring with unity. If 1 is of additive order n then ch R = n and if 1 is of additive order infinity then ch R is 0.

**3.3.2 Theorem**: If D is an integral domain, then characteristic of D is either zero or a prime number.

**3.3.3** Definition and examples of Nilpotent, Idempotent, product of rings.

**3.4** Definition and examples of Ideal

**3.4.1** Definition of Sum of two ideals. Examples.

**3.4.2 Theorem**: If A and B of two ideals of R then A + B is an ideal of R containing both A and B.

**3.5** Definition of Simple Ring.

**3.5.1 Theorem**: A division ring is a simple ring.

**Unit – 4 HOMOMORPHISM AND IMBEDDING OF RING**

**Lectures 10**

**4.1** Definition and examples of Quotient Rings, Homomorphism, Kernel of homomorphism.

**4.1.1 Theorem**: If $f : R \to R'$ is a homomorphism then f(0) = 0′.

**4.1.2 Theorem**: If $f : R \to R'$ is a homomorphism then f( –a) = – f(a).

**4.1.3 Theorem**: If $f : R \to R'$ is homomorphism then Ker f is an ideal of R.

**4.1.4 Theorem**: If $f : R \to R'$ is homomorphism then Ker f = {0} if and only if f is one –one.

**4.1.5 Fundamental Theorem of ring homomorphism**: If $f : R \to R'$ is an onto homomorphism with K = Ker f, then $\frac{R}{K} \cong R'$

**4.1.6 First Theorem of isomorphism**: Let B $\subseteq$ A be two ideal of ring R. Then $\frac{R}{A} \cong \frac{R/B}{A/B}$

**4.1.7 Second Theorem of isomorphism**: Let A, B be two ideals of ring R then $\frac{A+B}{A} \cong \frac{B}{A \cap B}$

**4.2** Definition of Imbedding ring.

**4.2.1 Theorem**: Any ring can be imbedded into a ring with unity.

**4.3** Definition and examples of Maximal Ideal and Prime ideal.

**4.3.1 Theorem**: Let R be a commutative ring with unity. An ideal M of R is maximal ideal of R iff $\frac{R}{M}$ is a field.

**4.3.2 Theorem**: Let R be a commutative ring. An ideal P of R is prime iff $\frac{R}{P}$ is an integral domain.

**REFERENCE BOOKS:**


1. A Course in Abstract Algebra**,**Vijay K. Khanna, S. K. Bhambri; Vikas Publishing House Pvt. Ltd., New - Delhi – 110014, Fourth Revised Edition 2013.
2. Topics in Algebra, HersteinI. N.; Vikas Publishing House, 1979.
3. Fundamentals of Abstract Algebra**,** Malik D. S. Morderson J. N. and Sen M. K. McGraw Hill,1997.
4. A Text Book of Modern Abstract Algebra, Shanti Narayan
5. Modern Algebra, Surjeet Sing and QuaziZameeruddin;Vikas Publishing House,1991.
6. Lectures on Abstract Algebra, T. M. Karade, J. N. Salunkhe, K. S. Adhav, M. S. Bendre, Sonu Nilu , Einstein Foundation International, Nagpur 440022.
7. Basic Algebra Vol. I & II , N. Jacobson ,W. H. Freeman 1980.
8. Algebra, VivekSaha I and VikasBistNarosa Publishing House, 1197.
9. A First Course in Abstract Algebra by John B. Fraleigh Pearson Education; Seventh edition (2014).

# Paper No. XI

# PARTIAL DIFFERENTIAL EQUATIONS

**Unit: 1 LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF DRDER ONE** **10 Lectures**

**1.1** Explanation of the terms :

**1.1.1** Partial differential Equation.

**1.1.2** Order of the Partial differential equation.

**1.1.3** Degree of the Partial differential equation

**1.2** Linear Partial Differential equation.

**1.2.1** Derivation of a partial differential equation by the elimination of arbitrary constants.

**1.2.2** Derivation of a partial differential equation by the elimination of arbitrary function & from the equation f(u, v) = 0 where u and v are the functions of x, y and z.

**1.2.3** Lagrange's Linear Partial Differential Equation.

**1.2.4** Lagrange's method of solving the linear partial differential equation Pp + Qq = R of order one.

**1.2.5** Working Rule for Solution of Langranges linear Partial differential equation Pp + Qq = R where P, Q, R are functions of x, y and z.

**1.3** Geometrical Interpretation of Langranges linear partial differential equation.

**Unit 2 NON-LINEAR PARTIAL DIFFERENTIAL EQUATIONS OF ORDER ONE** **10 Lectures**

**2.1** Explanation of the terms.

1. Non linear partial differential equation.
2. Solution or Integral of a partial differential equation.
3. Complete Integral.
4. Particular Integral.
5. General Integral.
6. Singular Integral.

**2.2** Special Methods of Solutions applicable to some standard forms.

**2.2.1 Standard Form I** : Partial differential Equations of the form $f(p, q) = 0$.

**2.2.2 Standard Form II** : Clairauts Form $Z = px + qy + f(p,q) = 0$.

**2.2.3 Standard Form III** : Partial differential Equations of the form $f(z, p, q) = 0$.

**2.2.4 Standard Form IV**:Partial differential Equations of the for $f_1(x, p) = f_2(y, q)$.

**2.3** General Method of Solving equations of order one but of any degree :

Charpit's Method.

**2.4** Working Rule for Charpit's Method.

**Unit 3 LINEAR HOMOGENEOUS PARTIAL DIFFERENTIAL EQATIONS WITH CONSTANT COEFFICIENTS. 12 Lectures**

**3.1** Explanation of the terms

1. Linear partial differential equation of order n.
2. Solution of Linear Partial differential equation.
3. Linear Homogeneous Partial differential equation with constant coefficients.

**3.2** Solution of linear homogeneous partial differential equation with constant coefficients of the form $F(D,D') \, z = f(x,y)$ where $F(D,D')$ is a homogeneous function of D and $D'$.

**3.3** Methods for finding the complementary functions (C.F).

**3.4** Methods for finding the particular Integrals (P.I).

**3.5** Finding the particular methods for Integral when f (x, y) is of the form

$(ax + by)$ when $F(a,b) = 0$ and $F(a,b) \neq 0$.

**3.6** General method for finding particular Integral.

**Unit 4 NON- HOMOGENEOUS LINEAR PARTIAL DIFFERENTIAL EQUATIONS WITH CONSTANT COEFFICIENTS 13 Lectures**

**4.1** Explanation of the terms:

1. Non – homogeneous linear partial differential equation with constant coefficients.

2. Solution of non homogeneous partial differential equation of the form $F(D, D')\,z = f(x, y)$ where $F(D, D')$ is non homogeneous function of D and $D^1$.

**4.2** Solution of the equation $(D - m D' - K)\,z = 0$

**4.3** Methods for finding the complementary function (C.F) of a non homogeneous equation $F(D, D')\,z = 0$ where $F(D, D')$ can be factorized in to linear factors of D and $D'$.

**4.4** Methods for finding particular Integral (P.I) of non homogeneous linear equations with constant coefficients of the form $F(D, D')\,z = f(x, y)$.

**Case I** : When $f(x, y) = e^{ax + by}$ and $F(a, b) = 0$

**Case II** : When $f(x, y) = \text{Sin}(ax + b)$ or $\text{Cos}(ax + b)$

**Case III** : When $f(x, y) = x^m y^m$

**Case IV** : When $f(x, y) = V.\, e^{ax + by}$, where V is a function of x and y.

**4.5** Method for finding complementary function of a non homogeneous equation $F(D, D')\,z = 0$ where $F(D, D')$ can not be factorized in to linear factors of D and $D^1$.

**4.6** Equations reducible to linear form with Constant coefficients.

**REFERENCE BOOKS:**

1.Defferential Equations : Gupta – Malic – Mittal. Pragati Prakashan Meerut.

2.Differential Equations : Sharma – Gupta, Krishna Prakashan Meerut.

3.Ordinary and Partial disserential equations: M.D. Raisinghnia S.Chand and Co. New Delhi.

# Paper XII
# Numerical Methods- I

**Unit- 1 NON-LINEAR EQUATIONS** **13 Lectures**

**1.1** Introduction: Polynomial equations, algebraic equations and their roots, iterative methods.

**1.2** Bisection method: algorithm, examples.

**1.3** Secant method: iterative sequence of secant method, examples.

**1.4** Regula-Falsi method: algorithm, graphical representation, examples.

**1.5** Newton's method: algorithm, examples.

**Unit- 2 SYSTEM OF LINEAR EQUATIONS: EXACT METHODS** **8 Lectures**

**2.1** Introduction: System of linear equations as a vector equation Ax = b, Augmented matrix

**2.2** Direct methods:

**2.2.1** Gauss elimination method: Procedure, examples.

**2.2.2** Gauss-Jordan method: Procedure, examples.

**2.3** Iterative methods: General iterative rule X(m + 1) = BX(m) + c .

**Unit- 3 SYSTEM OF LINEAR EQUATIONS: ITERATIVE METHOD** **10 Lectures**

**3.1** Jacobi iteration scheme X(m + 1) = BX(m) + c (Textbook page 3.41), examples.

**3.2** Gauss-Seidel method: Formula, examples.

**Unit- 4 EIGENVALUES ANA EIGENVECTORS** **14 Lectures**

**4.1** Eigenvalues and eigenvectors of a real matrix.

**4.2** Power method for finding an eigenvalue of greatest modulus.

**4.2.1** The case of matrix whose “dominant eigenvalue is not repeated”, examples.

**4.2.2** Method of exhaustion, examples.

**4.2.3** Method of reduction, examples.

**4.2.4** Shifting of the eigen value, examples.

## RECOMMENDED BOOK:

1. An Introduction to Numerical Analysis (Third Edition), Devi Prasad, Narosa Publishing House.

## REFERENCE BOOKS:

1. Introductory Methods of Numerical Analysis, S. S. Sastry, Prentice Hall of India.
2. Numerical Methods for Mathematics, Science and Engineering, J. H. Mathews, Prentice Hall of India.
3. Numerical Methods for Scientists and Engineers, K. Sankara Rao, Prentice Hall of India.
4. Numerical Analysis, Bhupendra Singh, Pragati Prakashan.

# SEMESTER – VI

# Paper – XIII

# METRIC SPACES

**UNIT –1 <u>LIMITS AND METRIC SPACES</u>** **8 lectures**

**1.1 Revision: Limits of a function on the real line**.

**1. 2 Metric space:** Definition :of Metric space and examples inclusive of each of $R^1, R_d, R^n, l^\infty, l^2$.

**1.3 Limits in metric spaces**

**1.3.1** Definition of $\lim_{x\to a} f(x) = L$.

**1.3.2** If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = N$ then

(i) $\lim_{x\to a}[f(x)+g(x)] = L+N$;

(ii) $\lim_{x\to a}[f(x)-g(x)] = L-N$;

(iii) $\lim_{x\to a}[f(x)g(x)] = LN$ and

(iv) $\lim_{x\to a}[f(x)/g(x)] = L/N \quad (N \neq 0)$

**5.4** Definition: Sequences and their convergence in metric space, Cauchy sequence in metric space.

**1.4.1** Theorem : A sequence of points in any metric space cannot converge to two distinct limits

**1.4.2** Theorem : Every convergent sequence in metric space is Cauchy.

**1.4.3** Example to illustrate that every Cauchy sequence need not be convergent.

1.4.3 Theorem : Every Cauchy sequence of real numbers is bounded.

1.4.4 Theorem : If a Cauchy sequence has a convergent subsequence then the sequence itself is convergent.

1.4.5 Theorem : Every Cauchy sequence in $R_d$ is convergent.

**UNIT - 2: <u>CONTINUOUS FUNCTIONS ON METRIC SPACES</u> 15 lectures**

2.1 Functions continuous at a point on the real line.

2.1.1 Definition: Continuity of a function

2.1.2 Theorem (Statement only) : If real valued functions $f$ and $g$ are continuous at $a \in R^1$, then so are

$f+g, \quad f-g, \quad fg, \quad f/g, \quad f \circ g, \quad cf, \quad |f|$ where, $c \in R$ at $a$.

2.2 Reformulation:

2.2.1 Theorem : The real valued function $f$ is continuous at $a \in R^1$ if and only if given $\varepsilon > 0$ there exists $\delta > 0$ such that

$$|f(x) - f(a)| < \varepsilon \qquad (|x-a| < \delta).$$

2.2.2 Definition: The open ball of radius *r* about *a*.

2.2.3 Theorem : The real valued function $f$ is continuous at $a \in R^1$ if and only if the inverse image under $f$ of any open ball $B[f(a);\varepsilon]$ about $f(a)$ contains an open ball $B[a;\delta]$ about *a*.

2.2.4 Theorem : A function $f$ is continuous at $a$, if and only if

$$\lim_{n\to\infty} x_n = a \Rightarrow \lim_{n\to\infty} f(x_n) = f(a).$$

2.3 Functions continuous on a metric space

2.3.1 Definition: The open ball of radius *r* about *a* in a metric space.

2.3.2 Definition: Continuity of function defined on a metric space

2.3.3 Theorem : The function $f$ is continuous at $a \in M_1$ if and only if any one of the following conditions hold

(i) Given $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\rho_2[f(x), f(a)] < \varepsilon \qquad (\rho_1(x,a) < \delta).$$

(ii) The inverse image under $f$ of any open ball $B[f(a);\varepsilon]$ about $f(a)$ contains an open ball $B[a;\delta]$ about $a$.

(iii) Whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence of points in $M_1$ converging to $a$, then the sequence $\{f(x_n)\}_{n=1}^{\infty}$ of points in $M_2$ converging to $f(a)$.

2.3.4 Theorem : If $f$ is continuous at $a \in M_1$ and $g$ is continuous at $f(a) \in M_2$, then $g \circ f$ is continuous at $a$.

2.3.5 Theorem : Let $M$ be a metric space, and let $f$ and $g$ be real valued functions which are continuous at $a \in M$, then so are $f+g,\quad f-g,\quad fg,\quad f/g,\quad |f|$ at $a$.

2.3.6 Definition of continuity of a function $f: M_1 \to M_2$.

2.3.7 Theorem : If $f$ and $g$ be continuous functions from a metric space $M_1$ into a metric space $M_2$, then so are $f+g,\quad f-g,\quad fg,\quad f/g,\quad |f|$ on $M_1$.

2.4 Open sets.

2.4.1 Definition: Open set

2.4.2 Any open ball in a metric space is an open set.

2.4.3 Theorem : In any metric space $<M,\rho>$, both $M$ and $\varnothing$ are open sets.

2.4.4 Theorem : Arbitrary union of open sets is open.

2.4.5 Theorem : Every subset of $R_d$ is open.

2.4.6 Theorem : Finite intersection open sets is open

2.4.7 Theorem : Every open subset $G$ of $R^1$ can be written as $G = \bigcup I_n$ where $I_1, I_2, \ldots$ are a finite number or a countable number of open intervals which are mutually disjoint.

2.4.8 Theorem : A function is continuous if and only if inverse image of every open set is open.

2.5 Closed sets

2.5.1 Definition: Limit point, closure of a set.

2.5.2 Theorem : If E is any subset of the metric space M, then $E \subset \overline{E}$.

2.5.3 Definition: Closed set.

2.5.4 Theorem : Let $E$ be a subset of the metric space $M$. Then the point $x \in M$ is a limit point of $E$ if and only if every open ball $B[x;r]$ about $x$ contains at least one point of $E$.

2.5.5 Theorem : Let $E$ be a subset of the metric space $M$, then $\overline{E}$ is closed.

2.5.6 Theorem : In any metric space $<M, \rho>$, both $M$ and $\varnothing$ are closed sets.

2.5.7 Theorem : Arbitrary intersection of closed sets is closed.

2.5.8 Theorem : Finite union of closed sets is closed.

2.5.9 Theorem :Let $G$ be an open subset of the metric space $M$. Then $G' = M - G$ is closed. Conversely, if $F$ is a closed subset of $M$, then $F' = M - F$ is open.

2.5.10 Theorem : Let $<M_1, \rho_1>$ and $<M_2, \rho_2>$ be metric spaces., and let $f : M_1 \to M_2$. Then $f$ is continuous on $M_1$ if and only if $f^{-1}(F)$ is a closed subset of $M_1$ whenever $F$ is a closed subset of $M_2$.

2.5.11 Theorem :Let $f$ be a 1-1 function from a metric space $M_1$ onto a metric space $M_2$. Then if $f$ has any one of the following properties, it has them all.

(i) Both $f$ and $f^{-1}$ are continuous (on $M_1$ and $M_2$, respectively).

(ii) The set $G_1 \subset M_1$ is open if and only if its image $f(G) \subset M_2$ is open.

(iii) The set $F \subset M_1$ is closed if and only if its image $f(F)$ is closed.

2.5.12 Definition : Homeomorphism, dense subset of a metric space.

2.5.13 Show that $R^1$ and $R_d$ are not homeomorphic.

**UNIT – 3 <u>CONNECTEDNESS, COMPLETENESS, AND COMPACTNESS</u>**

**15 lectures**

3.1 More about open sets

3.1.1 Theorem : Let $<M,\rho>$ be a metric space and let $A$ be a proper subset of $M$. Then the subset $G_A$ of $A$ is an open subset of $<A,\rho>$ if and only if there exists an open subset $G_M$ of $<M,\rho>$ such that $G_A = A \cap G_M$.

3.2 Connected sets:

3.2.1 Theorem : Let $<M,\rho>$ be a metric space and let $A$ be a subset of $M$. Then if $A$ has either one of the following properties it has the other.

(a) It is impossible to find nonempty subsets $A_1, A_2$ of $M$ such that

$$A = A_1 \cup A_2,\ \overline{A_1} \cap A_2 = \emptyset, A_1 \cap \overline{A_2} = \emptyset.$$

(b) When $<A,\rho>$ is itself regarded as metric space, then there is no set except $A$ and $\emptyset$ which is both open and closed in $<A,\rho>$.

3.2.2 Definition: Connected set

3.2.3 Theorem : The subset $A$ of $R^1$ is connected if and only if whenever $a \in A, b \in A$ with $a < b$, then $c \in A$ for any $c$ such that $a < c < b$.

3.2.4 Theorem : A continuous function carries connected sets to connected sets.

3.2.5 Theorem : If $f$ is a continuous real valued function on the closed bounded interval $[a,b]$, then $f$ takes on every value between $f(a)$ and $f(b)$.

3.2.6 Theorem : A metric space is connected if and only if every continuous characteristic function on it is constant.

3.2.7 Theorem : If $A_1$ and $A_2$ are connected subsets of a metric space $M$, and if $A_1 \cap A_2 \neq \emptyset$, then $A_1 \cup A_2$ is also connected.

3.2.8 Theorem : The interval $[0,1]$ is not connected sunset of $R_d$.

3.3 Bounded and totally bounded sets

3.3.1 Definition: Bounded subset of metric space, totally bounded sets.

3.3.2 Theorem : Every totally bounded set is bounded.

3.3.3 Theorem : A subset $A$ of $R_d$ is totally bounded if and only if $A$ contains only a finite number of points.

3.3.4 Definition: $\varepsilon-$dense set.

3.3.5 Theorem : The subset $A$ of the metric space $<M,\rho>$ is totally bounded if and only if , for every $\varepsilon > 0$, $A$ contains a finite subset $\{x_1, x_2, \ldots, x_n\}$ which is $\varepsilon-$dense in $A$.

3.3.6 Theorem : Let $<M,\rho>$ be a metric space. The subset $A$ of $M$ is totally bounded if and only if every sequence of points of $A$ contains a Cauchy sequence.

3.4 Complete metric space.

3.4.1 Definition: Complete metric space.

3.4.2 Theorem : If $<M,\rho>$ be a complete metric space, and $A$ is a closed subset of $M$, then $<A,\rho>$ is also complete.

3.4.3 Generalized nested interval theorem.

3.4.4 Definition: Contraction operator.

3.4.5 Theorem : Let $<M,\rho>$ be a complete metric space. If $T$ is a contraction on $M$, then there is one and only one point $x$ in $M$ such that $Tx = x$.

3.4.6 $R_d$ is complete and $R^2$ is complete.

3.5 Compact metric spaces

3.5.1 Definition: Compact metric space.

3.5.2 The metric space $<M,\rho>$ is compact if and only if every sequence of points in $M$ has a subsequence converging to a point in $M$.

3.5.3 Theorem : A closed subset of a compact metric space is compact.

3.5.4 Theorem : Every compact subset of a metric space is closed.

3.5.5 Definition: Covering and open covering

3.5.6 Theorem : If M is a compact metric space, then M has the Heine-Borel property.

3.5.7 Theorem : If a metric space M has Heine-Borel property, the M is compact.

3.5.8 Definition: Finite intersection property.

3.5.9 Theorem : The metric space $M$ is compact if and only if, whenever F is a family of closed subsets of $M$ with finite intersection property, then $\bigcap_{F\in\mathcal{F}} F \neq \varnothing$.

3.5.10 Theorem : Finite subset of any metric space is compact.

## UNIT- 4 <u>SOME PROPERTIES OF CONTINUOUS FUNCTIONS ON METRIC SPACE</u> **7 lectures**

4.1 Continuous functions on compact metric space

4.1.1 Theorem : Let $f$ be a continuous function of the compact metric space $M_1$ into a metric space $M_2$ . Then the range $f(M_2)$ of $f$ is also compact.

4.1.2 Theorem : Let $f$ be a continuous function of the compact metric space $M_1$ into a metric space $M_2$. Then the range $f(M_2)$ of $f$ is a bounded subset of $M_2$.

4.1.3 Definition: Bounded function

4.1.4 Theorem : If the real valued function $f$ is continuous on a closed bounded interval in $R^1$, then $f$ must be bounded.

4.1.5 Theorem : If the real valued function $f$ is continuous on the compact metric space $M$, then $f$ attains a maximum value at some point of $M$. Also, $f$ attains a minimum value at some point of $M$.

4.1.6 Theorem : If the real valued function $f$ is continuous on a closed bounded interval $[a,b]$, then $f$ attains a maximum and minimum value at some point of $[a,b]$.

4.1.7 Theorem : If $f$ is a continuous real valued function on the compact connected metric space $M$, then $f$ takes on every value between its minimum value and its maximum value.

4.2 Uniform continuity

4.2.1 Definition: Uniform continuity.

4.2.2 Let $< M_1, \rho_1 >$ be a compact metric space. If $f$ is a continuous function from $M_1$ into a metric space $< M_2, \rho_2 >$, then $f$ is uniformly continuous on $M_1$.

4.3.3 If the real valued function $f$ is continuous on the closed bounded interval $[a,b]$, then $f$ is uniformly continuous on $[a,b]$.

4.3.4 Let $< M_1, \rho_1 >$ be a metric space and let $A$ be a dense subset of $M_1$. If $f$ is a uniformly continuous function from $< A, \rho_1 >$ into a complete Metric space $< M_2, \rho_2 >$, then $f$ can be extended to a uniformly continuous function $F$ from $M_1$ into $M_2$.

## REFERENCE BOOKS


1. R. R. Goldberg, **Methods of Real Analysis,** Oxford and IBH Publishing House.

**2.** T. M. Apostol, **Mathematical Analysis,** Narosa Publishing House

**3.** Satish Shirali, H. L. Vasudeva, **Mathematical Analysis,** Narosa Publishing House.

**4.** D. Somasundaram, B. Choudhary, **First Course in Mathematical Analysis,** Narosa Publishing House

**5.** W. Rudin, **Principles of Mathematical Analysis,** McGraw Hill Book Company.

**6**. Shantinarayan, Mittal, **A Course of Mathematical Analysis,** S.Chand and Company.

**7.** J.N. Sharma, **Mathematical Analysis-**I, Krishna Prakashan Mandir, Meerut.

**8.** Malik, Arrora, **Mathematical Analysis,** Wiley Eastern Ltd.

**9.** Shantinarayan, M.D.Raisinghania, **Mathematical Analysis,** S.Chand

# Paper – XIV

# LINEAR ALGEBRA

**Unit –1: VECTOR SPACES:** **18 Lectures**

**1.1** Definition of vector space and simple examples.

**1.2 Theorem**: In any vector space V(F) the following results hold:

(i) 0.x = 0.

(ii) $\alpha$.0 = 0.

(iii) (-$\alpha$) x = - ($\alpha$x) = $\alpha$ (-x).

(iv) ( – β) x = $\alpha$x – β x .

**1.3** Definition of subspace and examples.

**1.4 Theorem**: A necessary and sufficient condition for a non empty subset W of a vector space V(F) to be a subspace is that W is closed under addition and scalar multiplication.

**1.5 Theorem**: A non empty subset W of a vector space V(F) is a subspace of V if and only if $\alpha x + \beta y \in W$ $for\, \alpha, \beta \in F, x, y \in W$ .

**1.6** Definition of sum of subspaces, direct sum, and quotient space. Examples.

Homomorphism of vector space ( Linear transformations)and examples.

**1.7 Theorem**: Under a homomorphism $T : V \to U$

i) T(0) = 0

ii) T(-x) = -T(x)

**1.8** Definition of Kernel and Range of homomorphism. Examples.

**1.9 Theorem** : Let $T : V \to U$ be a homomorphism, then Ker T is a subspace of V.

**1.10 Theorem**: Let $T : V \to U$ be a homomorphism, then Ker T = {0} if and only if T is one – one.

**1.11 Theorem**: Let $T : V \to U$ be a L.T. (linear transformation) then range of T is a subspace of U

**1.12 Theorem**: Let W be a subspace of V, then there exists an onto L.T.

$\theta : V \to \frac{V}{W}$ such that Ker $\theta$=W.

**1.13** Definition of Linear Span. Examples.

**1.14 Theorem**: L(S) is the smallest subspace of V containing S.

**1.15 Theorem**: If W is subspace of V then L(W) = W and conversely.

**1.16** Definition of Finite dimensional vector space ( F. D.V. S ), Linear dependence and independence, basis of vector space and examples.

**1.17 Theorem**: If S = $\{v_1, v_2, v_3, . . . ,v_n\}$is a basis of V then every element of V can be expressed uniquely as a linear combination of $v_1, v_2, v_3, . . . , v_n$.

**1.18 Theorem**: Suppose S is a finite subset of a vector space V such that V = L(S) then there exists a subset of S which is a basis of V.

**1.19** Definition of F.D.V.S. and examples.

**1.20 Theorem**: If V is a F.D.V.S. and $\{v_1, v_2, v_3, . . . ,v_r\}$is a L.I. subset of V, then it can be extended to form a basis of V.

**1.21 Theorem**: If dim V= n and S=$\{v_1, v_2, v_3, . . . ,v_n\}$ spans V then S is a basis of V.

**1.22 Theorem**: If dim V = n and S = $\{v_1, v_2, v_3, . . . ,v_n\}$is a L.I. subset of V, then S is a basis of V.

**Unit –2 LINEAR TRANSFORMATION** **10 Lectures**

**2.1** Definition of L.T., Rank,Nullity and Examples.

**2.2 Theorem** : A L.T. $T : V \to V$ is one – one iff T is onto.

**2.3 Theorem** : Let V and W be two vector spaces over F. Let $\{v_1, v_2, v_3, . . . ,v_n\}$ be a basis of V and $w_1, w_2, w_3, . . ., w_n$ be any vectors in W (not essentially distinct). Then there exists a unique L.T. $T : V \to W \; s.t. T(V_i) = w_i \;\; i = 1,2,...n$

**2.4 Theorem:** (Sylvester's Law): Suppose V and W are finite dimensional vector spaces over a field F.Let T:V→W be a linear transformation. Then rank T + nullity T = dim V.

**2.5 Theorem**: If $T:V \to V$ be a L.T.,then the following statements are equivalent.

i) Range T $\cap$ Ker T = {0}

ii) If T(T(v)) = 0 then T(v) = 0, v $\in$V.

**2.6** Definition of Sum and Product of L.T. Linear operator, Linear functional, examples.

**2.7 Theorem**: Let T, $T_1$, $T_2$, $T_3$ be linear operators on V and let $I:V \to V$ be the identity mapping I(v) = v for all v then

i) I T = T I = T

ii) $T(T_1 + T_2) = TT_1 + TT_2$ , $(T_1 + T_2)T = T_1T + T_2T$

iii) $(T_1 T_2) = (\alpha T_1) T_2 = T_1 (\alpha T_2)$.

iv) $T_1 (T_2 T_3) = (T_1 T_2) T_3$.

**2.8** Definition of Invertible L.T. and examples.

**2.9 Theorem**: A L.T. $T:V \to W$ is a nonsingular iff T carries each L.T. subset of V onto a L.I. subset of W.

**2.10 Theorem**: Let $T:V \to W$ be a L.T. where V and W are F.D.V.S. with same dimension. Then the following statements are equivalent:

i) T is invertible.

ii) T is nonsingular.

iii) T is onto.

**2.11 Theorem**: Let $T:V \to W$ and $S:W \to U$ be two L.T. Then

i) If S and T are one – one onto then ST is one-one onto and $(ST)^{-1} = T^{-1}S^{-1}$.

ii) If ST is one – one then T is one-one.

iii) If ST is onto then S is onto.

**2.12** Definition of Matrix of L.T. and examples.

**2.12.1.Theorem :** $Hom(U,V) \cong M_{m\times n}(F)$.

**2.13 Definition of Dual space.**

**2.13.1 Theorem:** Let V be an n dimensional vector space over a field F. Then the dimension the dual space of V over F is n.

**Unit –3 INNER PRODUCT SPACES** **10 Lectures**

**3.1** Definition of Inner product space, norm of a vector and examples.

**3.2 Theorem: Cauchy- Schwarz inequality.** Let V be an inner product space. Then $|(u, v)| \le \|u\| \, \|v\|$, for all $u, v \in V$.

**3.3 Theorem: Triangle inequality.** Let V be an inner product space. Then $\| u + v \| \le \| u \| + \| v \|$, for all $u, v \in V$.

**3.4 Theorem: Parallelogram law.** Let V be an inner product space. Then $\| u + v \|^2 + \| u - v \|^2 = 2 (\| u \|^2 + \| v \|^2)$, for all $u, v \in V$.

**3.5** Definition of Orthogonal vectors and orthonormal sets.

**3.6 Theorem**: Let S be a orthogonal set of non-zero vectors in an inner product space V. Then S is a linearly independent set.

**3.7** Gram-Schmidt orthogonalisation process

**3.7.1 Theorem:** Let V be a non trivial inner product space of dimension n. Then V has an orthonormal basis.

**3.7.2 Examples.**

**Unit – 4 EIGEN VALUES AND EIGEN VECTORS** **7 Lectures**

**4.1** Definition of Eigen values, Eigen vectors, Eigen space of order n.

**4.1.1** Examples.

**4.2.1 Theorem**: Let T be a linear operator on a finite dimensional vector space V over F. Then $c \in F$ is an eigen value of T if and only if $T - cI$ is singular.

**4.2.2 Thoerm**: Let dim V = n. Let T be a linear operator on V. Let $v_1, v_2, \ldots, v_k$ be eigen vectors of T, corresponding to distinct eigen values $c_1, c_2, \ldots, c_k$ of T. Then $v_1, v_2, \ldots, v_k$ are linearly independent.

**4.2.3 Theorem**: Suppose every non zero vector of a FDVS is an eigen vector of a linear operator T on V. Then T is a scalar multiple of I.

**4.2.4 Theorem**: Let v and w be eigen vectors of T corresponding to two distinct eigen values of T . Then v + w cannot be an eigen vector of T

**4.2.5** Let V be a two dimensional vector space over the field R of real numbers. Let T be a linear operator on V such that $T(v_1) = \alpha v_1 + \beta v_2$, $T(v_2) = \gamma v_1 + \delta v_2$, $\alpha, \beta, \gamma, \delta \in R$, where $\{ v_1, v_2 \}$ is a basis of V. Then find necessary and sufficient condition that 0 be a characteristic root of T.

**4.2** Charateristic Polynomials

**4.2.1** Definition of Charateristic Polynomials.

**4.2.2 Theorem**: Similar matrices have same characteristic polynomial.

**4.2.3 Theorem**: Let $c \neq 0$ be an eigen value of an invertible operator T. Then $c^{-1}$ is an eigen value of $T^{-1}$.

**4.2.4 Theorem**: Let A be a real $n \times n$ matrix. Let $\lambda$ be a real eigen value of A. Then there exists an eigen vector X of A corresponding to eigen value $\lambda$ such that X is also real.

**4.3** Charateristic polynomial of a Linear operator and remarks on it.

**4.4** Examples on eigen values and eigen vectors

## REFERENCE BOOKS


1. **A Course of Abstract Algebra,** Khanna V. K. and Bhambri M. S. ,4th edition Vikas Publishing House PVT Ltd., New Delhi , 2013.
2. **Linear Algebra,** Holfman K. and Kunze R. Pentice Hall of India, 1978.
3. **Linear Algebra,** Lipschutz' s,Schaum's Outline Series, McGraw Hill, Singpore, 1981.
4. **Matrix and Linear Algebra,** K. B. Datta, Prentice Hall of India Pvt. Ltd. New Delhi 2000.
5. **University Algebra,**N.S. Gopalkrishnan , Wiley Eastern Limited , First Reprint May 1988.
6. **A Text Book of Modern Algebra, R**. Balakrishnan.

# Paper – XV

# Complex Analysis

**Unit-1 ANALYTIC FUNCTION** **13 Lectures**

**1.1** limit continuity of a function of a complex variable.

**1.2** complex valued function.

**1.3** Differentiability and continuity and elementary rules of Differentiation.

**1.4** Analytic function and Analytic function in domain.

**1.5** Necessary and sufficient condition for F(z) = u+iv to be Analytic and examples.

**1.6** Limit of a sequence of complex numbers.

**1.7** Polar form of Cauchy- Riemann Equation.

**1.8** Harmonic function, conjugate harmonic function.

**1.9** Construction of Analytic function.

**1.10** Solved problem s related to the test of analyticity of functions and construction of analytic function.

**Unit-2 COMPLEX INTEGRATION** **17 Lectures**

**2.1** Elementary Definitions.

**2.2** Complex line integral.

Integral along oriented curve and examples

**2.3** Cauchy's integral theorem and its consequences,

Cauchy's integral formula for multiply connected domain and its examples.

**2.4** Jordan curve, orientation of Jordan curve

**2.5** Simple connected and multiply connected domain.

**2.6** Rectifiable curve and their properties.

**2.7** Properties

(a) $\int_c [f(z) + g(z)]dz - \int_c f(z)dz + \int_c g(z)dz$ ]

(b) If C has two parts C1 and $C_2$ then

$$\int_c f(z)\,dz = \int_{C1} f(z)\,dz + \int_{C2} f(z)\,dz$$

(c) $\int_c f(z)\,dz = \int_{C1} f(z)\,dz + \int_{C2} f(z)\,dz$

**2.8** Higher order derivative of an analytic function.

**2.9** Development of an analytic function as a power series

(a) Taylor's theorem for complex function.

(b) Examples on Taylor's and Laurent series.

**Unit -3 SINGULARITIES AND RESIDUES** **8 Lectures**

**3.1** Zeros of an analytic function, singular point.

**3.2** Different types of singularity, poles and zeros are isolated.

**3.3** Limiting point of zeros and poles.

**3.4** Residue theorem, residue at a pole and residue at infinity.

**3.5** Cauchy's residue theorem

**3.6** Computation of residue at a finite pole

**3.7** Integration round unite circle

**3.8** Jordan's lemma

**3.9** Evaluation of Integrals $\int_{-\infty}^{+\infty} f(z)dz$ when f(z) has no poles on the real line when poles on the real line.

**Unit-4 ENTIRE MEROMORPHIC FUNCTIONS** **7 Lectures**

**4.1** Definition of entire and meromorphic functions.

(a) Characterization of polynomials as entire functions

(b) Characterization of rational functions as meromorphic functions.

**4.2** Mittag Leffler's expansion Rouche's theorem and solved problems.

**4.3** Some theorems on poles and singularities

## REFERENCE BOOKS


1) Functions of a complex variables.-Dr. J.K.Goyal , K.P.Gupta , Pragati Prakashan meerut post post box no 62 Meerut-1 (U.P.)

2) Complex Variables-J.N.Sharma, Krishna Prakashan mandir Subhsh Bazar Meerut-2(U.P.)

3) Complex Variable- Shanti Narayan , P.K.Mittal, S Chand And Company Ltd Ram nager , New Delhi 110055

4) Complex Analysis(an introduction course) - G.R. Viswanat South Corolina State University ( U.S.A)

5) Complex Analysis-Join H. Mathews, Russa W. H.,Narosa Publishing House, Ne

# Paper –XVI
# Numerical Methods- II

**Unit- 1 INTERPOLATION:EQUAL INTERVALS** **10 Lectures**

**1.1** Forward interpolation:

**1.1.1** Newton's forward differences, forward difference table

**1.1.2** Newton's forward form of interpolating polynomial (formula only), examples

**1.2** Backward interpolation:

**1.2.1** Newton's backward differences, backward difference table, Newton's backward form of interpolating polynomial (formula only), examples

**Unit- 2 INTERPOLATION:UNEQUAL INTERVALS** **08 Lectures**

**2.1** Introduction, Lagrangian interpolating polynomial (formula only), examples

**2.2** Divided difference interpolation:

**2.2.1** Newton's divided differences, divided difference table, examples (finding divided (differences of given data)

**2.2.2** Newton's divided difference form of interpolating polynomial, examples

**Unit- 3- NUMERICAL DIFFERENTIATION AND INTEGRATION**
**17 Lectures**

**3.1** Numerical differentiation based on interpolation polynomial.

**3.2** Numerical integration:

**3.2.1** Newton-Cotes formula (statement only)

**3.2.2** Basic Trapezoidal rule (excluding the computation of error term), composite Trapezoidal rule, examples.

**3.2.3** Basic Simpson's $1/3^{rd}$ rule (excluding the computation of error term), composite Simpson's $1/3^{rd}$ rule, examples.

**3.2.4** Basic Simpson's $3/8^{th}$ rule (excluding the computation of error term), composite Simpson's $3/8^{th}$ rule, examples.

**Unit- 4 ORDINARY DIFFERENTIAL EQUATIONS** **10 Lectures**

**4.1** Euler method, examples

**4.2** Second order Runge-Kutta method (formula only), examples

**4.3** Fourth order Runge-Kutta method (formula only), examples

## RECOMMENDED BOOK

An Introduction to Numerical Analysis (Third Edition), Devi Prasad, Narosa Publishing House.

## REFERENCE BOOKS

**1** Introductory Methods of Numerical Analysis, S. S. Sastry, Prentice Hall of India.

**2** Numerical Methods for Mathematics, Science and Engineering, J. H. Mathews, Prentice Hall of India.

**3** Numerical Methods for Scientists and Engineers, K. Sankara Rao, Prentice Hall of India.

**4** Numerical Analysis, Bhupendra Singh, Pragati Prakashan.

**REVISED SYLLABUS OF B. Sc. Part III MATHEMATICS (Practical)**

**Implemented from June – 2015**

# Computational Mathematics Laboratory - IV
# (Operations Research Techniques)

| Sr.No. | Topic | No.of Practicals |
|---|---|---|
| | Linear Programming | |
| 1 | Simplex Method : Maximization Case | 1 |
| 2 | Simplex Method : Minimization Case | 1 |
| 3 | Two-Phase Method | 1 |
| 4 | Big-M-Method | 1 |
| | Transportation Problems | |
| 5 | North- West Corner Method | 1 |
| 6 | Least Cost Method | 1 |
| 7 | Vogel’s Approximation Method | 1 |
| 8 | Optimization of T.P. by Modi Method | 1 |
| | Assignment Problems | |
| 9 | Hungarian Method | 1 |
| 10 | Maximization Case in Assignment Problem | 1 |
| 11 | Unbalanced Assignment Problems | 1 |
| 12 | Traveling Salesman Problem | 1 |
| | Theory of Games | |
| 13 | Games with saddle point | 1 |
| 14 | Games without saddle point : (Algebraic | 1 |
| 15 | method) | |
| | Games without saddle point: | |
| | a) Arithmetic Method | 1 |
| 16 | b) Matrix Method | |
| | Games without saddle point : Graphical | |
| | method | 1 |

**REFERENCE BOOKS:**

1) Operations Research [Theory and Applications], By J.K.Sharma, Second edition, 2003, Macmillan India Ltd., New Delhi

2) Operations Research: S. D. Sharma.

# Computational Mathematics Laboratory –V
# (Numerical Methods)

| Sr. No. | Title | No. of Practicals |
|---|---|---|
| 1 | Bisection method | 1 |
| 2 | Secant method | 1 |
| 3 | Newton's method | 1 |
| 4 | Gauss elimination method | 1 |
| 5 | Gauss-Jordan method | 1 |
| 6 | Jacobi iteration scheme | 1 |
| 7 | Gauss-Seidel method | 1 |
| 8 | Power method | 1 |
| 9 | Newton's forward interpolation | 1 |
| 10 | Newton's backward interpolation | 1 |
| 11 | Lagrangian interpolation | 1 |
| 12 | Divided difference interpolation | |
| 13 | Trapezoidal rule | 1 |
| 14 | Simpson's $1/3^{rd}$ rule | 1 |
| 15 | Second order Runge-Kutta method | 1 |
| 16 | Fourth order Runge-Kutta method | 1 |

**RECOMMENDED BOOKS:**


1. An Introduction to Numerical Analysis (Third Edition), Devi Prasad, Narosa Publishing House.
2. Introductory Methods of Numerical Analysis, S. S. Sastry, Prentice Hall of India.
3. Numerical Methods for Mathematics, Science and Engineering, J. H. Mathews, Prentice Hall of India.
4. Numerical Methods for Scientists and Engineers, K. Sankara Rao, Prentice Hall of India.
5. Numerical Analysis, Bhupendra Singh, Pragati Prakashan.

# Computational Mathematics Laboratory – VI
# Numerical Recipes in C++, Scilab

| Sr.No. | Topic | No. Of Practicals |
|---|---|---|
| 1 | **<u>C++ -Introduction</u> : History, Identifiers, Keywords, constants, variables, C++ operations.** | **1** |
| 2 | **<u>Data typesin C++</u>: Integer, float, character. Input/Output statements, Header files in C++, iostreanm.h, iomanip.h,math.h.** | |
| 3 | **<u>Expressions in C++</u> : (i) constant expression, (ii) integer expression, (iii) float expression, (iv) relational expression,(v) logical expression, (vi) Bitwise expression. <u>Declarations in C++</u>.**<br><br>**<u>Program Structure of C++</u> .**<br>**Simple program to " WEL COME TO C++ ".** | **1** |
| 4 | **<u>Control Statements</u>:**<br>**(a) if, if – else, nested if.**<br>**(b) for loop, while loop, do-while loop.**<br>**(c) break, continue, goto, switch statements.**<br>***Euclid's algorithm to find gcd and then to find lcm of two numbers a, b**<br>*** To list 1!, 2!, 3!, … , n! .**<br>*** To print prime numbers from 2 to n.** | **1** |
| 5 | **<u>Arrays</u> : (a) Sorting of an array.**<br>**(b) Linear search.**<br>**(c) Binary search.**<br>**(d) Reversing string.** | **1** |
| 6 | **<u>Functions</u> : User defined functions of four types with illustrative programs each.** | **1** |
| 7 | **<u>Numerical Methods to find roots of a given function</u> :**<br>**(a) Bisection function.**<br>**(b) Newton – Raphson Method.** | **1** |
| 8 | **<u>Interpolation</u> :**<br>**(a) Lagrange's interpolation formula.**<br>**(b) Newton Gregory forward interpolation formula.**<br>**(c) Newton Gregory backward interpolation formula.** | **1** |

| | | |
|---|---|---|
| **9** | **<u>Numerical</u> <u>Methods</u> <u>for</u> <u>solution</u> <u>of a system of</u> <u>Linear</u> <u>Equations</u>:**<br>**( Unique solution case only )**<br>**(a) Gauss – Elimination Method.**<br>**(b) Gauss – Jorden Method.** | **1** |
| **10** | **<u>Numerical</u> <u>Methods</u> <u>for</u> <u>solution</u> <u>of Ordinary</u> <u>Differential</u> <u>Equations</u>:**<br>**(a) Euler Method**<br>**(b) Euler′s Modified Method**<br>**(c) Runge Kutta Second and Fourth order Method.** | **2** |
| **11** | **<u>Computation with Scilab</u>**<br>**(The Students is expected to familiarize with Scilab software for numerical computations)**<br>**1) Basics of Scilab: Introduction, creating real variables, elementary mathematical functions.**<br>**2) Creating matrix in Scilab, accessing elements in matrix.**<br>**3) Empty matrix, the colon “:” operator, the “eye” matrix, the dollar “$” operator, element wise Operations.**<br>**4) Using Scilab functions to find the inverse of matrix, trace of matrix,**<br>**5) Determinant of square matrix, addition and multiplication of matrices, eigen values and eigen vectors.**<br>**6) Conditional statements and loops: the “if” statement, the “for” loop**<br>**7) Defining and using functions in scilab**<br>**8) Plotting : Creating graphs of simple functions.** | **6** |


1. **Programming with C++, D. Ravichandran Second Edition, Tata Mac- Graw-Hill publishing Co. Ltd., New Delhi (2006).**

2. **Introduction to Scilab, Michael Baudin**

# Computational Mathematics Laboratory – VII
# PROJECT, STUDY- TOUR, VIVA – VOCE

**A : PROJECT** **[30 Marks]**

Each student of B.Sc. III is expected to read, collect, understand the culture of Mathematics, its historic development. He is expected to get acquainted with Mathematical concepts, innovations, relevance of Mathematics. Report of the project work should be submitted through the respective Department of Mathematics.

**Topics for Project work:**

**1**.Contribution of the great Mathematicians such as Rene Descart, Leibnitz, Issac Newton, Euler, Lagrange, Gauss, Riemann, Fourier, Bhaskaracharya, Srinivas Ramanujan etc.

**2**. On the following topics or any other equivalent topic:

(i) Theorem on Pythagoras and Pythagorean triplets.

(ii) On the determination of value of π.

(iii) Remarkable curves.

(iv) Orthogonal Latin Spheres.

(v) Different kinds of numbers.

(vi) Law of quadratic reciprocity of congruence due to Gauss.

(vii) Invention of Zero.

(viii) Vedic Mathematics.

(ix) Location of objects in the celestial sphere.

(x) Kaprekar or like numbers

(xi) Playing with PASCAL TRIANGLE and FIBONACCI NUMBERS.

(xii) CPM and PERT

(xiii) Magic squares.

(xiv) Software such as Mupad, Matlab, Mathematica, Xplore.

(xv) Fuzzy Mathematics.

(xvi) Applications of Fuzzy Mathematics.

(xvii) Pigeon hole principle

(xviii) Mathematics For All.

(xix) Recent Trends in Mathematics.

(xx) Rough sets and Soft sets.

Evaluation of the project report will be done by the external examiners at the time of annual examination.

**B. STUDY TOUR** **[05 Marks]**

**List of suggested places:**

Banglore, Goa (Science Center), Pune, Kolhapur, Mumbai, Ahamdabad, Hydrabad, etc.

**C. VIVA-VOCE (on the project report).** **[15Marks]**

## REFERENCE BOOKS


1. The World of Mathematics, James R. Newman & Schuster, New York.

2. Men Of Mathematics, E.T.Bell.

3. Ancient Indian Mathematics, C. N. Srinivasayengar.

4. Vedic Mathematic , Ramanand Bharati.

5. Fascinating World of Mathematical Science Vol. I, II, J. N. Kapur.


## JOURNALS


1. Mathematical Education.

2. Mathematics Today.

3. Bona Mathematical.

4. Ramanujan Mathematics News Letter.

5.Resonance.

6.Mathematical Science Trust Society (MSTS), New friends colony, New Delhi 4000065.

# Nature of Question papers (Theory)

**COMMON NATURE OF QUESTION FOR THEORY PAPER MENTIONED SPERATELY:**

**Nature of Practical Question Paper**

(1) **COMPUTATIONAL MATHEMATICS LABORATRY - IV**

**This carries** 50 **marks.**

**Examination : 40 Marks**

**Journal : 10 Marks**

(2) **COMPUTATIONAL MATHEMATICS LABORATRY - V**

**This carries** 50 **marks.**

**Examination : 40 Marks**

**Journal : 10 Marks**

(3) **COMPUTATIONAL MATHEMATICS LABORATRY - VI**

**This carries** 50 **marks.**

**Examination : 40 Marks**

**Journal : 10 Marks**

(4) **COMPUTATIONAL MATHEMATICS LABORATRY - VII**

**This carries** 50 **marks.**

**Project : 30 Marks ( External Examiner)**

**Study Tour : 05 Marks ( External Examiner)**
**Viva Voce : 15 Marks ( External Examiner)**

**Note : Each student of a class will select separate topic for project work He/ She should submit the reports of his / her project work , Study tour report to the department and get the same certified.**

**Teaching Periods :**

(i) Total teaching periods for Paper – V , VI, VII, VIII are **12** (Twelve) per week. 3 (Three) periods per paper per week.

(ii) Total teaching periods for Computational Mathematics Laboratory – III, IV, V, VII for the whole class are **20** **(Twenty)** per week. 5 (Five) periods per Lab. per week.